Question 1
Find the continued fraction of $\sqrt{12}$ and $\sqrt{17}$.

Question 2
Find the quadratic numbers that belong to the continued fractions

$[3, 1, 6, 1, 6, 1, 6, 1, 6, ...]$ and $[2, 1, 8, 1, 8, 1, 8, 1, 8, ....]$.

Question 3
Show that for any natural numbers $p, q \in \mathbb{N}$ one has

$$\left| \sqrt{5} - \frac{p}{q} \right| > \frac{1}{5q^2}.$$ 

Question 4
Find at least two different solutions to the equation

$$1 + 2 + \ldots + k = (k + 1) + (k + 2) + \ldots + (l - 1) + l,$$

with $k, l \in \mathbb{N}$ and $l > k$ and show how it is related to a Pell’s equation.

Note: A simple non-programmable calculator is allowed for the exam.