2ND EXAM ‘INLEIDING IN DE GETALTHEORIE’

Tuesday, 11th October 2016, 9 am - 10 am

Question 1
Compute the following symbols

\[
\left( \frac{71}{97} \right), \left( \frac{53}{97} \right), \left( \frac{137}{227} \right), \left( \frac{10}{5} \right)
\]

Question 2
Let \( p \) be an odd prime number. In the lectures we have seen that \( \left( \frac{-1}{p} \right) = 1 \) if and only if \( p \equiv 1 \mod 4 \). Characterize in the same way all prime numbers with \( \left( \frac{5}{p} \right) = 1 \).

Question 3
Let \( p \) be an odd prime number and \( q \) the smallest quadratic non residue modulo \( p \). More precisely, let \( q \) be the smallest natural number which is not a quadratic residue modulo \( p \). Prove that \( q \) is a prime number.

Question 4
Show that the sequence \( n^5 - n + 3 \), with \( n \in \mathbb{N} \) does not contain any squares.

Note: A simple non-programmable calculator is allowed for the exam. (If you don’t have one, don’t worry, you don’t necessarily need it.)