Problem 1 (15 points).  
(a) Give an example of a space $X$ with two points $x, y \in X$ such that $\pi_1(X, x)$ is not isomorphic to $\pi_1(X, y)$.

(b) Give an example of a surface (i.e. a 2-dimensional compact connected manifold without boundary) that is not homeomorphic to $S^2$, the torus, the Klein bottle or $\mathbb{R}P^2$ and give its fundamental group. (You can give the result without proof.)

(c) Draw a triangulation of $\mathbb{R}P^2$. 
Continuation of Problem 1
Problem 2 (8 points). Let $A \subseteq X$ be a subspace of a space $X$ and $F : I \times X \to X$ a map such that

- $F(0, x) = x$, $\forall x \in X$,
- $F(t, x) \in A$, $\forall x \in A$, $t \in I$,
- $F(1, x) \in A$, $\forall x \in X$.

Show that $A$ and $X$ are homotopy equivalent.
Problem 3 (15 points). Consider two disjoint embeddings $f, g: D^2 \rightarrow S^1 \times S^1$ into the torus. Let $X$ be $S^1 \times S^1 \setminus (f(D^2) \cup g(D^2))$. Show that the fundamental group of $X$ is isomorphic to $\mathbb{Z} \ast \mathbb{Z} \ast \mathbb{Z}$. You can choose $f$ and $g$ as you please for this purpose.
Continuation of Problem 3
Problem 4 (22 points). Let $A = \{ z \in \mathbb{C} | 1 \leq |z| \leq 2 \} \subset \mathbb{C}$.

(a) Let $f : A \rightarrow A$ be a homeomorphism. Show that $f(z) \in \partial A$ if $z \in \partial A$ and $f(z) \notin \partial A$ if $z \notin \partial A$.

Hint: You are allowed to use the result from the homework that no point in $\partial D^2$ has a neighborhood in $D^2$ that is homeomorphic to $\mathbb{R}^2$.

(b) Denote for points $x, y \in \mathbb{C}$ the line from $x$ to $y$ by $L_{x,y}$. Define $B = A \cup L_{0,1} \cup L_{2,3}$ and $C = A \cup L_{0,1} \cup L_{2i,3i}$. Let furthermore $X$ be as in the previous problem the torus with two disks removed. Which of the spaces $A, B, C$ and $X$ are homeomorphic and which are homotopy equivalent?
Continuation of Problem 4