3RD EXAM 'INLEIDING IN DE GETALTHEORIE'

Tuesday, 23rd October 2018, 9:30 am - 10:30 am

Question 1 (4 points)
Find the continued fraction expansion of $\sqrt{15}$ and $\sqrt{26}$.

Question 2 (4 points)
Find the quadratic numbers that belong to the continued fractions

$$\langle 2,2,4,2,4,2,4,2,4,\ldots\rangle \quad \text{and} \quad \langle 4,1,8,1,8,1,8,1,8,\ldots\rangle.$$

Question 3 (4 points)
Show that there are infinitely many natural numbers which cannot be written as the sum of at most 15 fourth powers. In other words, show that there are infinitely many $n \in \mathbb{N}$ such that the equation

$$n = x_1^4 + x_2^4 + \ldots + x_{15}^4$$

has no solutions with $x_1, \ldots, x_{15} \in \mathbb{Z}$.

Question 4 (4 points)
Let N be a natural number which is not equal to a square of a natural number. Let

$$\sqrt{N} = \langle a_0, \overline{a_1, \ldots, a_k}\rangle$$

its continued fraction expansion, with k minimal. Show that

$$k \leq 2N - 1.$$

For this you may use all lemmas and propositions and theorems that we discussed in the lectures.

Hint: a quadratic number in normal form $\frac{P+\sqrt{D}}{Q}$ is reduced iff $0 < P < \sqrt{D}$ and $\sqrt{D} - P < Q < \sqrt{D} + P$.

Note: A simple non-programmable calculator is allowed for the exam.

Date: 23rd October 2018.