Tentamen Inleiding Topologie, WISB243 2019-01-29, 13:30 – 16:30

- Write your **name** on every sheet, and on the first sheet your **student number**, **group** (1: Aldo and Francesco, 2: Maarten) and the total **number of sheets** handed in.
- Use a **separate sheet** for each exercise!
- You may use the lecture notes, the extra notes and personal notes, but no worked exercises.
- Do not just give answers, but also justify them with complete arguments. If you use results from the lecture notes, always **mention this**, and show that their hypotheses are fulfilled in the situation at hand.
- **N.B.** If you fail to solve an item within an exercise, **do continue**; you may then use the information stated earlier.
- The weights by which exercises and their items count are indicated in the margin. The highest possible total score is 44. The exam grade E will be obtained from your total score T by rounding off $\min(T/4, 10)$ to one decimal accuracy.
- You are free to write the solutions either in English, or in Dutch.

Succes !

13 pt total **Exercise 1.** For $\mathbb{R}$ we consider the collection $\mathscr{B}$ of all subsets of the form

$$(a,n] := \{x \in \mathbb{R} \mid a < x \leq n\},$$

with $n \in \mathbb{Z}$ and $a \in \mathbb{R}$ such that $a < n$.

2 pt (a) Show that $\mathscr{B}$ is not a topology, but it is a topology basis. Denote by $\mathscr{T}$ the topology generated by $\mathscr{B}$.

4 pt (b) Is $(\mathbb{R}, \mathscr{T})$ second countable? Is it Hausdorff? Is it metrizable?

Consider the set $A := [\,\frac{1}{2}\,, 1\,]$.

3 pt (c) Determine the closure of A.

4 pt (d) Show that A is compact, and that $[0,1]$ is not compact.

10 pt total **Exercise 2.**

1 pt (a) Show that $\mathbb{R}^2 \setminus \{(0,0)\}$ is connected for the topology induced by the Euclidean topology.

4 pt (b) Show that there exists no continuous injective map $f : \mathbb{R}^2 \to \mathbb{R}$.

5 pt (c) If M is a two-dimensional topological manifold, show that there exists no continuous injective map $g : M \to S^1$. Hint: use (b).

11 pt total **Exercise 3.** Let X be topological space, and $A \subset X$ a subset. The space X/A obtained by collapsing A to a point is equipped with the quotient topology. The quotient map is denoted by $\pi : X \to X/A$.

1 pt (a) Show that $\pi^{-1}(\pi(A)) = A$.

2 pt (b) If $S \subset X - A$ show that $\pi^{-1}(\pi(S)) = S$.

2 pt (c) If $T \subset X$ satisfies $T \supset A$, show that $\pi^{-1}(\pi(T)) = T$.

From now on we assume that A is connected for the induced topology.

4 pt (d) If X is the union of two disjoint non-empty open subsets U and V, show that $\pi(U)$ and $\pi(V)$ are disjoint open subsets of X/A.

2 pt (e) Show that X is connected if and only if X/A is connected.

10 pt total **Exercise 4.** Let X be a paracompact topological space and A a closed subset of X. The goal of this exercise is to show that A is paracompact for the induced topology. For this, we assume that $\mathscr{V} = \{V_i\}_{i \in I}$ is an open covering of A.

1 pt (a) Show that there exists a family $\mathscr{U} = \{U_i\}_{i \in I}$ of open subsets of X such that $V_i = U_i \cap A$ for all $i \in I$.

2 pt (b) Show that $\mathscr{U}^* := \mathscr{U} \cup \{X - A\}$ is an open covering of X.

1 pt (c) Show that there exists a locally finite refinement $\mathscr{W} = \{W_j \mid j \in J\}$ of $\mathscr{U}^*$.

3 pt (d) Let $J_0 = \{j \in J \mid W_j \subset X - A\}$ and put $J_1 = J - J_0$. Show that $\mathscr{W}' = \{W_j \cap A\}_{j \in J_1}$ is an open covering of A which is subordinate to $\mathscr{V}$.

3 pt (e) Show that A is paracompact.