Utrecht University

Utrecht University School of Economics

Endterm exam Econometrics (WISB377)

Thursday, 10 November 2022, 11:00-13:00 CET

For those who have permission from the Board of Examiners for an extension of time, they can hand in the answer sheet on 13:40 CET ultimately.

Exam instructions

At the start of the exam

- Candidates who arrive 30 minutes after the time scheduled for the start of the examination will not be permitted entry to the examination room.

During the examination

- Nobody is allowed to leave the room within the first 30 minutes after the start of the exam.
- You are not allowed to go to the restroom unless you have permission of the Examiner or an invigilator.
- **MOBILE PHONES AND OTHER COMMUNICATION DEVICES ARE ONLY ALLOWED WHEN SWITCHED OFF AND STORED IN CLOSED BAGS**.
- *It is a closed book exam. It is **not** allowed to use any study aids such as books, readers, (preprogrammed) calculators*
- You may use a simple calculator and a dictionary (without any [handwritten] notes in it).
- The exam form is **NOT** allowed to be taken home by the candidate

Results/Post-examination regulations:

- The results of the examination will be announced on Blackboard within two weeks of the exam date. At the same time the time & place of the exam inspection will be announced.
- We do not discuss exam results over the phone or by email.
- After the announcement of the exam results in OSIRIS you have four weeks within which to lodge an appeal against your grade.
- Four weeks after the results of this exam are published, the original exam is available to you, when a declaration is signed, stating that no appeal has been made or will be made.

You can request a photocopy of your answers at the Student Desk up and until four weeks after publication of the results

Questions. In total 9 points.

a) (1 point) For the OLS estimator $\hat{\boldsymbol{\beta}}$ and the five assumptions

Assumption 1: $\text{rank}(\mathbf{X}) = k+1$.

Assumption 2: A linear population regression equation,

$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{u}$, for which the variables in $\mathbf{X}$ are random variables.

Assumption 3: Strict exogeneity $E(\mathbf{u} \mid \mathbf{X}) = \mathbf{0}$.

Assumption 4: The error terms are homoskedastic and they are

independently distributed $E(\mathbf{u}\mathbf{u}' \mid \mathbf{X}) = \sigma_u^2 \mathbf{I}_n$.

Assumption 5: $\mathbf{u} \mid \mathbf{X} \sim Normal(\mathbf{0}, \sigma_u^2 \mathbf{I}_n)$, an n-dimensional multivariate normal

distribution with expected value $\mathbf{0}$ and covariance matrix $\sigma_u^2 \mathbf{I}_n$.

Question: could you please derive the test statistic for

$$\mathbf{R}\boldsymbol{\beta} = \mathbf{r}$$

R: q x (k+1) matrix, which can be used for testing. $\mathbf{r}$ is a q-dimensional vector (q is the number of restrictions)

Please make clear how the assumptions are used.

Question: Why do we emphasize that the test statistic is valid only under H0?

b) (1 point) For a sample of size n, if (what you don't need to prove)

1. $\frac{1}{n}\mathbf{X}'\mathbf{X} \xrightarrow{p} \mathbf{C}$ for which $\mathbf{C}$ is finite and the inverse of $\mathbf{C}$ exists.

2. $\frac{1}{\sqrt{n}}\sum_{i=1}^{n}\mathbf{x}_i u_i \xrightarrow{d} Normal(\mathbf{0}, \sigma_u^2 \mathbf{C})$

could you please apply the Central Limit Theorem for the OLS estimator $\hat{\boldsymbol{\beta}}_n$ to derive the statistical distribution of $\sqrt{n}(\hat{\boldsymbol{\beta}}_n - \boldsymbol{\beta})$

c) (1 point) For the linear population regression equation, $\mathbf{y} = \mathbf{X\beta} + \mathbf{u}$ for which

$$Var(\mathbf{u} \mid \mathbf{X}) = \mathbf{\Psi} = E(\mathbf{uu}' \mid \mathbf{X}) = \begin{pmatrix} \sigma_1^2 & 0 & \cdots & 0 \\ 0 & \sigma_2^2 & & 0 \\ \vdots & & \ddots & \\ 0 & & & \sigma_n^2 \end{pmatrix}$$

and

$$Var(\hat{\boldsymbol{\beta}} \mid \mathbf{X}) = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{\Psi}\mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}$$

could you please derive the estimation procedure for robust standard errors? What are the assumptions?

d) (1 point) For the loss function $L(\boldsymbol{\beta}) = (\mathbf{y} - \mathbf{X\beta})'\Theta(\mathbf{y} - \mathbf{X\beta})$, for which Θ is an *n*x*n* matrix, please derive the corresponding estimator $\hat{\boldsymbol{\beta}}$ and demonstrate that $L(\boldsymbol{\beta})$ attains a minimum at $\hat{\boldsymbol{\beta}}$. Can the estimator be characterized as a GLS estimator? Please motivate your answer.

e) (2 points) For the AR(2) model

$$u_t = \rho_1 u_{t-1} + \rho_2 u_{t-2} + e_t \qquad t = 3,\ldots,T$$

where e_t: i.i.d. (identically and independently distributed) with $Ee_t = 0$; $Var(e_t) = \sigma_e^2$
e_t is uncorrelated to u_{t-1} and u_{t-2}
<u>Question</u>: please derive the covariance matrix $Var(\mathbf{u} \mid \mathbf{X}) = \mathbf{\Psi}$. Are there any restrictions to the size of the parameters ρ_1 and ρ_2?

f) (3 points) For the specification

$$y_{it} = \mathbf{x}_{it}'\boldsymbol{\beta} + \alpha_i + u_{it} \qquad i = 1,\ldots,n; t = 1,\ldots,T$$

1. For a random-effects specification, please derive the matrix $\mathbf{\Psi}_i = Var(\alpha_i + u_{it})$

2. How would you test for fixed-effects estimator versus a first-differences estimator? Please derive the test statistic and outline the testing procedure. What is the zero and the alternative hypothesis?
3. Please derive the Hausman test statistic. What is the zero and the alternative hypothesis?

< end of the exam >