

WISB272 GAME THEORY FINAL EXAM

- This is a **closed book** exam. However, you are **allowed to use a cheat sheet** while working on it. The cheat sheet has to be handwritten by you, and can have two two-sided sheets A4.
 - You have 3 hours to work on the exam (plus additional 30 min for students with extra time).
 - Please write your solutions in **English**.
 - Show your work on each problem. **All answers must be justified.** Mysterious or unsupported answers will not receive credit.
 - **If you use a theorem or proposition from class, clearly indicate this.**
 - **Organize your work**, in a reasonably neat and coherent way. Work scattered all over the page without a clear ordering will receive very little credit.
 - **Good luck!**
1. Consider the following zero-sum game, where the payoff matrix depends on the parameter x :

$$A(x) = \begin{pmatrix} 1 & 2-x & 2 \\ 2x+3 & 2 & 3 \\ 2 & 1-x & x^2 \end{pmatrix}.$$

- (a) (10 points) Find the value of the game and a pair of optimal strategies for players I and II in the cases when $x = 1$ and when $x = -1$.
- (b) (10 points) Now, suppose players I and II play according to the following rules. First, each of the players, independently of the other one, chooses a value of the parameter $x \in \{-1, 1\}$. If both players choose the same value of the parameter, they play the corresponding game; otherwise, the payoff for each of them is zero. Find the value of this game.
2. (10 points) Consider the following general-sum game for two players:

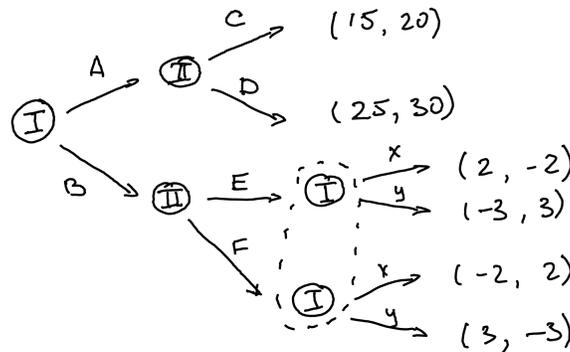
	A	B	C
a	$(-5, 3)$	$(0, 2)$	$(0, 5)$
b	$(5, 3)$	$(3, -3)$	$(0, 0)$
c	$(0, 0)$	$(3, 3)$	$(3, 5)$

Find all Nash equilibria for this game.

3. There are two firms producing the same product. Suppose they have different production costs: Firm I's cost of producing amount x of the product is $x + 2$, and Firm II's cost of producing amount y of the product is $y + 5$. Suppose also that the price-per-unit of the product is $P(x, y) = 17 - x - y$, where $x, y \geq 0$ are the amounts produced by I and II, respectively.

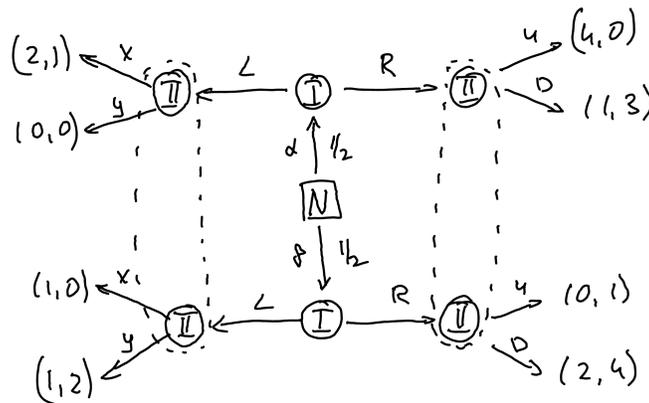
- (a) (10 points) Find all the pure Nash equilibria for this game and associated profits for firms.
- (b) (5 points) Are the Nash equilibria you found Pareto optimal?

4. Consider the following game in extensive form, given by the diagram below.



- (a) (5 points) Identify all subgames of this game and write this game in the normal form.
- (b) (10 points) Find all subgame perfect Nash equilibria, in pure and mixed strategies.

5. Consider the following game in extensive form, given by the diagram below.



- (a) (10 points) Find all separating perfect Bayesian equilibria in pure strategies.
- (b) (10 points) Find all pooling perfect Bayesian equilibria in pure strategies.

6. Consider a symmetric evolutionary game given by the following bi-matrix

	A	B
A	(5, 5)	(0, 1)
B	(1, 0)	(1, 1)

- (a) (10 points) Find all evolutionary stable strategies (ESS) for this game using the definition of ESS.
- (b) (10 points) Find all ESS for this game using replicator dynamics.