Utrecht University | Instructor: P. Salanevich
Block 2, 2021-22 | Date: April 19, 2022

WISB272 Game Theory
Final Exam Resit

- This is a **closed book** exam. However, you are **allowed to use a cheat sheet** while working on it. The cheat sheet has to be handwritten by you, and can have two two-sided sheets A4.
- You have 3 hours to work on the exam (plus additional 30 min for students with extra time).
- Please write your solutions in **English**.
- Show your work on each problem. **All answers must be justified.** Mysterious or unsupported answers will not receive credit.
- **If you use a theorem or proposition from class, clearly indicate this**.
- **Organize your work**, in a reasonably neat and coherent way. Work scattered all over the page without a clear ordering will receive very little credit.
- **Good luck!**

1. (10 points) Solve the following zero-sum game:

$$A = \begin{pmatrix} 9 & 5 & 2 & 1 \\ 8 & 4 & 6 & -2 \\ 0 & -2 & 4 & 6 \\ 3 & 2 & 2 & 3 \end{pmatrix}.$$

2. (a) (10 points) Find all Nash equilibria in the following general-sum game for two players:

	A	**B**	**C**	**D**
a	$(-3,-4)$	$(2,-1)$	$(0,6)$	$(1,1)$
b	$(2,0)$	$(2,2)$	$(3,0)$	$(1,-2)$
c	$(2,-3)$	$(-5,1)$	$(-1,-1)$	$(1,-3)$
d	$(-4,1)$	$(3,-5)$	$(1,2)$	$(-3,1)$

   (b) (10 points) Three firms (players I, II, and III) put three items on the market and advertise them either on morning or evening TV. A firm advertises exactly once per day. If more than one firm advertises at the same time, their profits are zero. If exactly one firm advertises in the morning, its profit is \$200K. If exactly one firm advertises in the evening, its profit is \$300K. Firms must take their advertising decisions simultaneously. Find a **symmetric** mixed Nash equilibrium.

3. (10 points) Consider the following Bertrand pricing game with brand loyalty and continuous pricing. Two pizza shops (Roma's and Tony's) are the only shops in a geographical market. Each firm faces marginal costs MC=3 and no fixed costs of production. Each selects a price (simultaneously) and knows that its demand q_i depends on both its own price P_i and its rival's price P_j as follows:

$$q_i = 12 + 0.5P_j - P_i.$$

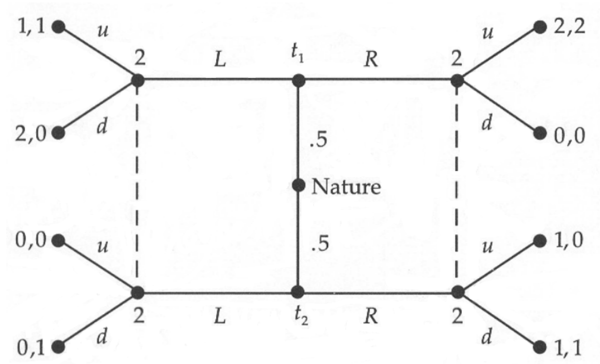
Find all the pure Nash equilibria for this game and associated profits for firms.

4. Consider the following voting process. Three legislators are voting on whether to give themselves a pay raise. The raise is worth b but each legislator who votes for the raise incurs a cost of voter resentment equal to $c < b$. The outcome is decided by the majority vote. The legislators vote sequentially and publicly (that is, 2 sees 1's vote and 3 sees both 1 and 2's votes). (b) Find a Nash equilibrium for this game using backward induction. Show that it is best to go first. (c) Show that there are two Nash equilibrium in which 3 votes no. Clearly specify these Nash equilibria. Why cant these equilibria be found by backward induction? Is there something strange about these equilibria?

   (a) (10 points) Write this game in the extensive form. Find all subgame perfect equilibria. Is it best to vote first, second, or third?

   (b) (5 points) Show that there are two Nash equilibria for this game in which the third legislator votes against the pay raise. Are these Nash equilibria subgame perfect?

5. Consider the following game in extensive form, given by the diagram below.



   (a) (10 points) Find all separating perfect Bayesian equilibria in pure strategies.

   (b) (10 points) Find all pooling perfect Bayesian equilibria in pure strategies.

6. Consider a symmetric evolutionary game given by the following bi-matrix

	A	**B**
A	$(2,2)$	$(4,5)$
B	$(5,4)$	$(1,1)$

   (a) (10 points) Find all evolutionary stable strategies (ESS) for this game using the definition of ESS.

   (b) (10 points) Find all ESS for this game using replicator dynamics.