Utrecht University

Utrecht University School of Economics

Midterm exam Econometrics (Wisb377)

Tuesday, 12 October 2021, 13:00-15:00 CET

For those who have permission from the Board of Examiners for an extension of time, they can hand in the answer sheet on 15:40 CET ultimately.

Exam instructions

At the start of the exam

• Candidates who arrive 30 minutes after the time scheduled for the start of the examination will not be permitted entry to the examination room.

During the examination

• Nobody is allowed to leave the room within the first 30 minutes after the start of the exam.
• You are not allowed to go to the restroom unless you have permission of the Examiner or an invigilator.

• MOBILE PHONES AND OTHER COMMUNICATION DEVICES ARE ONLY ALLOWED WHEN SWITCHED OFF AND STORED IN CLOSED BAGS.
• It is a closed book exam. It is not allowed to use any study aids such as books, readers, (preprogrammed) calculators
• You may use a simple calculator and a dictionary (without any [handwritten] notes in it).
• The exam form is NOT allowed to be taken home by the candidate

Results/Post-examination regulations:

• The results of the examination will be announced on Blackboard within two weeks of the exam date. At the same time the time & place of the exam inspection will be announced.
• We do not discuss exam results over the phone or by email.
• After the announcement of the exam results in OSIRIS you have four weeks within which to lodge an appeal against your grade.
• Four weeks after the results of this exam are published, the original exam is available to you, when a declaration is signed, stating that no appeal has been made or will be made.
You can request a photocopy of your answers at the Student Desk up and until four weeks after publication of the results

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This exam contains 8 subquestions (a – h)

Questions

a) After minimizing the quadratic loss function $L(\beta)$, the necessary first-order condition for the Ordinary Least Squares estimator $\hat{\beta}$ is

$$\nabla_p L(\beta) = \nabla_p y' y - \nabla_p 2y' X\beta + \nabla_p \beta' X' X\beta = 0$$

Question 1: Demonstrate that the OLS estimator $\hat{\beta}$ is a linear estimator in $y$.

Question 2: When is the OLS estimator $\hat{\beta}$ a unique estimator? Please motivate your answer.

b) For a sample of $n$ observations, a researcher wants to apply Ordinary Least Squares to estimate the regression parameters of the linear regression equation

$$\log(Wage_i) = \beta_0 + \beta_1 Edu_i + \beta_2 Exper_i + u_i \quad i = 1, ..., n$$

The researcher examines the assumptions that are required for a consistent estimator of the regression parameters of equation (1).

Question: in this setting stochastic independence plays a role in two different ways. Please explain carefully how.

c) Let’s assume that $\beta_1$ is small enough to apply the Taylor approximation

$$\log(1+x) \approx x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + ... \quad \text{as} \quad x \to 0$$

By making use of the Taylor approximation, could you please mathematically derive the formal interpretation of the parameter $\beta_1$ in equation (1)?

d) The variable Experience (labour market experience in years) is distinguished in four different categories. Each of them gets a 0-1 dummy variable. The specification becomes

$$\log(Wage_i) = \beta_0 + \beta_1 Edu_i + \gamma_1 Exper1_i + \gamma_2 Exper2_i + \gamma_3 Exper3_i + u_i$$

$Exper1 = 1$ if $0 \leq Experience < 5$ years

$= 0$ otherwise

$Exper2 = 1$ if $5 \leq Experience < 10$ years

$= 0$ otherwise
Exper3 = 1 if \( 10 \leq \text{Experience} < 15 \) years
= 0 otherwise
Exper4 = 1 if \( \text{Experience} \geq 15 \) years
= 0 otherwise

Question: Please give a careful interpretation of the parameter \( \gamma_i \) in equation (2).

e) Please proof the following property. Let \( \mathbf{y} \) be an \((n \times 1)\)-dimensional random vector. \( \mathbf{A}: (m \times n) \) non-random matrix; \( \mathbf{b}: (m \times 1) \)-non-random vector. Proof that \( \text{Var} (\mathbf{A} \mathbf{y} + \mathbf{b}) = \mathbf{A} \text{Var} (\mathbf{y}) \mathbf{A}^\top \)

f) Question: by deriving the covariance matrix of the OLS estimator, show the necessary assumptions that are required for \( \text{Var}(\hat{\mathbf{\beta}} \mid \mathbf{X}) = \sigma_u^2 (\mathbf{X}^\top \mathbf{X})^{-1} \).

So, it is insufficient to mention these assumptions only without any further derivation.

g) Could you please demonstrate that the covariance matrix of the OLS estimator \( \text{Var}(\hat{\mathbf{\beta}} \mid \mathbf{X}) = \sigma_u^2 (\mathbf{X}^\top \mathbf{X})^{-1} \) is a positive definite matrix?

h) Please proof the following:
Let \( \mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_n \) be a sequence of identically and independently distributed \((k+1)\) dimensional random variables, for which \( \mathbb{E} \mathbf{x}_i \mathbf{x}_i^\top = \mathbf{C} \). \( \mathbf{C} \) is a finite matrix for which the inverse exists. It is assumed that \( \sigma_u^2 \) is known.

\textbf{Result:} \( \sigma_u^2 \left( \sum_{i=1}^{n} \mathbf{x}_i \mathbf{x}_i^\top \right)^{-1} \xrightarrow{a.s.} \mathbf{O} \) (an \((k+1) \times (k+1)\) a matrix of zeros)

< end of the exam >