Utrecht University

Utrecht University School of Economics

Midterm exam Econometrics (Wisb377)

Tuesday, 12 October 2021, 13:00-15:00 CET

For those who have permission from the Board of Examiners for an extension of time, they can hand in the answer sheet on 15:40 CET ultimately.

Exam instructions

At the start of the exam

- Candidates who arrive 30 minutes after the time scheduled for the start of the examination will not be permitted entry to the examination room.

During the examination

- Nobody is allowed to leave the room within the first 30 minutes after the start of the exam.
- You are not allowed to go to the restroom unless you have permission of the Examiner or an invigilator.

- **MOBILE PHONES AND OTHER COMMUNICATION DEVICES ARE ONLY ALLOWED WHEN SWITCHED OFF AND STORED IN CLOSED BAGS**.
- *It is a closed book exam. It is **not** allowed to use any study aids such as books, readers, (preprogrammed) calculators*
- You may use a simple calculator and a dictionary (without any [handwritten] notes in it).
- The exam form is **NOT** allowed to be taken home by the candidate

Results/Post-examination regulations:

- The results of the examination will be announced on Blackboard within two weeks of the exam date. At the same time the time & place of the exam inspection will be announced.
- We do not discuss exam results over the phone or by email.
- After the announcement of the exam results in OSIRIS you have four weeks within which to lodge an appeal against your grade.
- Four weeks after the results of this exam are published, the original exam is available to you, when a declaration is signed, stating that no appeal has been made or will be made.

You can request a photocopy of your answers at the Student Desk up and until four weeks after publication of the results

This exam contains 8 subquestions (a – h)

Questions

a) After minimizing the quadratic loss function $L(\boldsymbol{\beta})$, the necessary first-order condition for the Ordinary Least Squares estimator $\hat{\boldsymbol{\beta}}$ is

$$\nabla_{\boldsymbol{\beta}} L(\boldsymbol{\beta}) = \nabla_{\boldsymbol{\beta}} \mathbf{y}'\mathbf{y} - \nabla_{\boldsymbol{\beta}} 2\mathbf{y}'\mathbf{X}\boldsymbol{\beta} + \nabla_{\boldsymbol{\beta}} \boldsymbol{\beta}'\mathbf{X}'\mathbf{X}\boldsymbol{\beta} = \mathbf{0}$$

Question 1: Demonstrate that the OLS estimator $\hat{\boldsymbol{\beta}}$ is a linear estimator in $\mathbf{y}$.

Question 2: When is the OLS estimator $\hat{\boldsymbol{\beta}}$ a unique estimator? Please motivate your answer.

b) For a sample of *n* observations, a researcher wants to apply Ordinary Least Squares to estimate the regression parameters of the linear regression equation

(1) $\log(Wage_i) = \beta_0 + \beta_1 Educ_i + \beta_2 Exper_i + u_i \qquad i = 1, \ldots, n$

The researcher examines the assumptions that are required for a consistent estimator of the regression parameters of equation (1).

Question: in this setting stochastic independence plays a role in two different ways. Please explain carefully how.

c) Let's assume that β_1 is small enough to apply the Taylor approximation

$$\log(1+x) \approx x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots \qquad \text{as } x \to 0$$

By making use of the Taylor approximation, could you please mathematically derive the formal interpretation of the parameter β_1 in equation (1)?

d) The variable *Experience* (labour market experience in years) is distinguished in four different categories. Each of them gets a 0-1 dummy variable. The specification becomes

(2) $\log(Wage_i) = \beta_0 + \beta_1 Educ_i + \gamma_1 Exper1_i + \gamma_2 Exper2_i + \gamma_3 Exper3_i + u_i$

$Exper1 = 1$ if $0 \leq Experience <$ 5 years
$= 0$ otherwise
$Exper2 = 1$ if $5 \leq Experience <$ 10 years
$= 0$ otherwise

$Exper3 = 1$ if 10 ≤ $Experience <$ 15 years

$= 0$ otherwise

$Exper4 = 1$ if $Experience \geq$ 15 years

$= 0$ otherwise

Question: Please give a careful interpretation of the parameter γ_1 in equation (2).

e) Please proof the following property. Let $\mathbf{y}$ be an (*n* x 1)-dimensional random vector. **A**: (*m* x *n*) non-random matrix; **b**: (*m* x 1)-non-random vector. Proof that $Var\left(\mathbf{Ay}+\mathbf{b}\right)=\mathbf{A}Var\left(\mathbf{y}\right)\mathbf{A}'$

f) Question: by deriving the covariance matrix of the OLS estimator, show the necessary assumptions that are required for $Var(\hat{\boldsymbol{\beta}} \mid \mathbf{X}) = \sigma_u^2(\mathbf{X}'\mathbf{X})^{-1}$.

So, it is insufficient to mention these assumptions only without any further derivation.

g) Could you please demonstrate that the covariance matrix of the OLS estimator $Var(\hat{\boldsymbol{\beta}} \mid \mathbf{X}) = \sigma_u^2(\mathbf{X}'\mathbf{X})^{-1}$ is a positive definite matrix?

h) Please proof the following:

Let $\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_n$ be a sequence of identically and independently distributed (*k*+1) dimensional random variables, for which $E\mathbf{x}_i\mathbf{x}_i' = \mathbf{C}$. $\mathbf{C}$ is a finite matrix for which the inverse exists. It is assumed that σ_u^2 is known.

Result: $\sigma_u^2\left(\sum_{i=1}^{n}\mathbf{x}_i\mathbf{x}_i'\right)^{-1} \xrightarrow{a.s.} \mathbf{O}$ (an (*k*+1) x (*k*+1) a matrix of zeros)

< end of the exam >