Course name: Econometrics (Wisb377)
Date examination: November 7, 2019
Duration 2.5 hours; from <10:00> to <12:30>
Examination: Endterm
Total number of pages: 4
Total number of exercises: 6

Full name: ---------------------------------------------------------------
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At the start of the exam
- Candidates who arrive 30 minutes after the time scheduled for the start of the examination will not be permitted entry to the examination room.

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- Nobody is allowed to leave the room within the first 30 minutes after the start of the exam.
- You are not allowed to go to the restroom unless you have permission of the Examiner or an invigilator.
- MOBILE PHONES AND OTHER COMMUNICATION DEVICES ARE ONLY ALLOWED WHEN SWITCHED OFF AND STORED IN CLOSED BAGS.
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- The exam form is NOT allowed to be taken home by the candidate

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Question 1)
For the OLS estimator \( \hat{\beta} = (X'X)^{-1} X'y \) of the linear regression equation \( y = X\beta + u \), for which \( y \) and \( u \) are \( n \)-dimensional vectors, \( \beta \) is a \((k+1)\)-dimensional vector, \( X \) is a \((n \times (k+1))\) dimensional matrix,

\[ u \mid X \sim \text{Normal}(0, \sigma_u^2 I_n) \]

with \( \sigma_u^2 \) nonzero and constant.

a) Please derive the statistical distribution of

\[ (3\hat{\beta} + 2\iota) \mid X \]

for which \( \iota \) is a \((k+1)\) dimensional vector of ones.

b) We proceed with \( u \mid X \). It is assumed that \( u \mid X \sim \text{Normal}(0, \Psi) \), for which \( \Psi \) is a symmetric matrix for which the inverse exists.

Question: Please derive the distribution of \( u'\Psi^{-1}u \mid X \)

Question 2)
For a random sample of \( n \) observations, the Ordinary Least Squares estimator \( \hat{\beta}_n \) is applied to the \((k+1)\)-dimensional vector of parameters \( \beta \) of the linear regression model

\[ y = X\beta + u \]

We consider the Central Limit Theorem, for which we formulate two additional assumptions.

1) \[ \frac{1}{n} X'X \overset{p}{\longrightarrow} C \] for which \( C \) is a finite and invertible \((k+1) \times (k+1)\) matrix.

2) \[ \frac{1}{\sqrt{n}} \sum_{i=1}^{n} x_i \mu_i \overset{d}{\longrightarrow} \text{Normal}(0, \sigma_u^2 C) \]

Furthermore, it can be shown that \( \sqrt{n} (\hat{\beta}_n - \beta) = \left( \frac{1}{n} X'X \right)^{-1} \frac{1}{\sqrt{n}} \sum_{i=1}^{n} x_i \mu_i \) (you don’t need to derive this expression).
a) Using the information of above, could you please derive the limiting statistical distribution of

\[ \sqrt{n} \left( \hat{\beta}_n - \beta \right) \]

**Question 3**
For a sample of \( n \) observations, for the linear regression model.

\[ y = X\beta + u \]

let’s assume that the variance covariance matrix of the error terms contains heteroskedasticity:

\[ \text{Var}(u | X) = \text{diag} \left( \sigma_i^2 \right) \quad i = 1, ..., n \]

a) Please, discuss the consequences of heteroskedasticity for the expected value of the OLS estimator \( \hat{\beta} = (X'X)^{-1}X'y \).

b) It can be demonstrated that \( \text{Var}(\hat{\beta} | X) = (X'X)^{-1}X'\Psi X(X'X)^{-1} \), for which

\[ X'\Psi X = \sum_{i=1}^{n} x_i \sigma_i^2 x_i' \]. Please carefully describe both stages of the estimation procedure to calculate the White robust standard errors.

**Question 4**
Let’s consider the linear regression model

\[ y = X\beta + u \]

for which the covariance matrix of the error terms is

\[ \Psi = \text{Var}(u | X) \]

a) Derive the Generalized Least Squares estimator \( \hat{\beta}_{GLS} \) by minimizing the loss function

\[ \min_{\beta} L(\beta) = \min_{\beta} (y - X\beta)'\Psi^{-1}(y - X\beta) \]
b) What assumptions are required for the existence of the GLS estimator $\hat{\beta}_{GLS}$?

**Question 5)**
We consider the AR(1) model

$$x_t = \rho x_{t-1} + e_t \quad |\rho| < 1 \quad t = 1, \ldots, T$$

where $e_t$: i.i.d. (identically and independently distributed) with $Ee_t = 0$; $\text{Var}(e_t) = \sigma_e^2$. $e_t$ is uncorrelated to $x_{t-1}$.

a) Please demonstrate that $\rho$ can be interpreted as the correlation between $x_t$ and $x_{t-1}$.

**Question 6)**
We consider the panel data model

$$y_{it} = \mathbf{x}_{it}' \mathbf{\beta} + \alpha_i + u_{it} \quad i = 1, \ldots, n; t = 1, \ldots, T$$

for which $\alpha_i$ is the individual-specific effect (random variable) with constant variance, and $u_{it}$ is the identically and independently distributed error term with expected value zero and constant variance.

a) Demonstrate that the assumption of strict exogeneity is required for the fixed-effects estimator.

b) Describe how the Mundlak’s formulation of the regression equation

$$y_{it} = \mathbf{x}_{it}' \mathbf{\beta} + \alpha_i + u_{it} \quad i = 1, \ldots, n; t = 1, \ldots, T$$

can be applied to test for a random effects specification (the zero hypothesis) versus fixed effects specification (the alternative hypothesis).

< end of the exam >