Course name: Econometrics (Wisb377)
Date examination: October 17, 2019
Duration 2 hours; from <11:00>, to <13:00>
Examination: Midterm
Total number of pages: 4
Total number of exercises: 3

Full name : -----------------------------------------------------------------------------------------------------------------------------------------------------------------
Student ID Number : -------------------------------------------------------------------------------------------------------------------------

Copy your Name and ID Number on every separate sheet/answering paper if required

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- Put your ID on your table for inspection.

Exam instructions
At the start of the exam
• Candidates who arrive 30 minutes after the time scheduled for the start of the examination will not be permitted entry to the examination room.

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• Nobody is allowed to leave the room within the first 30 minutes after the start of the exam.
• You are not allowed to go to the restroom unless you have permission of the Examiner or an invigilator.
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• You may use a simple calculator and a dictionary (without any [handwritten] notes in it).
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Questions

1) The vector of parameters $\beta$ of the linear regression model

$$y = X\beta + u$$

is estimated by Ordinary Least Squares (OLS), using a sample of $n$ observations.

$$\hat{\beta} = (X'X)^{-1}X'y$$

We are interested in the assumptions that are needed to derive an unbiased estimator

$$E(\hat{\beta} | X) = \beta.$$ 

a) Please give a careful explanation why it can be useful to have the assumption of a randomly drawn sample for an unbiased estimator. Motivate your answer by providing the proof for $E(\hat{\beta} | X) = \beta$.

b) Next, it is assumed that the variance-covariance matrix of the vector of error terms is

$$\text{Var}(u | X) = \sigma_u^2 I_n$$

Please derive the $(k+1) \times (k+1)$ variance-covariance matrix of $\hat{\beta}$.

c) For $\hat{y} = X(X'X)^{-1}X'y$ and $\hat{u} = y - \hat{y}$, demonstrate that

$$\hat{u}'\hat{u} = u'M_xu$$

for which $M_x = I_n - X(X'X)^{-1}X'$. 
2)
a) For the regression equation

\[ \text{wage}_i = \beta_0 + \beta_1 \text{birthyear}_i + \beta_2 \text{year}_i + \beta_3 \text{age}_i + u_i \quad i = 1, \ldots, n \]

there is perfect multicollinearity because

\[ \text{year}_i = \text{birthyear}_i + \text{age}_i \]

Please compute the \( R^2 \) of the auxiliary regression:

\[ \text{birthyear}_i = \alpha_0 + \alpha_1 \text{year}_i + \alpha_2 \text{age}_i + \upsilon_i \]

for which

\[ R^2 = 1 - \frac{\sum_{i=1}^{n} \hat{\upsilon}_i^2}{\sum_{i=1}^{n} (\text{birthyear}_i - \text{birthyear})^2} \]

b) The dependent variable \( y \) is regressed on a vector of ones (with no further explanatory variables)

\[ y_i = \beta + u_i \quad i = 1, \ldots, n \quad \text{with} \ Var(u) = \sigma_u^2 \]

Compute the following for this case

- The Ordinary Least Squares estimator \( \hat{\beta} \)
- Compute \( \hat{y}_i \) and \( \hat{u}_i \) \( i = 1, \ldots, n \)
- Compute \( \hat{\sigma}_u^2 \)
- Compute \( R^2 \)
- For \( \mathbf{M}_X = \mathbf{I}_n - \mathbf{X} (\mathbf{X}' \mathbf{X})^{-1} \mathbf{X}' \), compute \( \mathbf{M}_X \) for \( \mathbf{X} = \mathbf{t} \) and show that \( \mathbf{M}_i \) is a non-invertible, symmetric and idempotent matrix. Compute the trace of \( \mathbf{M}_i \).

c) For the linear regression equation

\[ \ln(\text{wage}_i) = \beta_0 + \beta_1 \text{education}_i + \beta_2 \text{age}_i + \beta_3 \text{age}_i^2 + u_i \]

compute the partial effect of age on the wage at \( \text{age} = 20 \). Please, give a careful motivation of the assumption(s), for which the partial effect of age on the dependent variable can be interpreted as a causal effect?
3) For the OLS estimator \( \hat{\beta} = (X'X)^{-1}X'y \) of the linear regression equation \( y = X\beta + u \), for which \( y \) and \( u \) are \( n \)-dimensional vectors, \( \beta \) is a \( (k+1) \)-dimensional vector, \( X \) is a \( n \times (k + 1) \) dimensional matrix,

c) Please give a careful description – included the assumptions – of the Strong Law of Large Numbers.

< end of the exam >