1. The stock price of company AB at time $t_0 = 0$ equals $S_0 = 10.0\text{€}$. A call option on this stock with exercise in 60 days and strike price $K = 10.5\text{€}$ costs $c(t_0) = 1\text{€}$. Prepare a table in which the stock price on the exercise date in the first row varies from 9.5€ to 12.5€. In the second row, put the profit/loss, in percentage, when buying the stock at $t_0$ and selling it at $t = T$; in the third row, the profit/loss at time $t = T$, in percentage, when buying the option at $t_0$. Compare the gains and losses in the option and stock returns.

<table>
<thead>
<tr>
<th>stock price at time $t = T$</th>
<th>9.5 €</th>
<th>10 €</th>
<th>10.5 €</th>
<th>11 €</th>
<th>11.5 €</th>
<th>12 €</th>
<th>12.5 €</th>
</tr>
</thead>
<tbody>
<tr>
<td>profit stock</td>
<td></td>
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<tr>
<td>profit option</td>
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</tbody>
</table>

2. Show that, for a continuously, differentiable function $g(t)$, the process

$$X(t) = g(t)W(t) - \int_0^t g'(z)W(z)dz,$$

is a martingale w.r.t. the natural filtration generated by the Brownian motion $W(t)$, where $g'(t)$ is the first derivative of $g(t)$, and subsequently show that

$$E[e^{2t}W(t)] = E \left[ \int_0^t 2e^{2z}W(z)dz \right].$$

3. A strangle, $S_t(t)$, is an option construction in which the investor takes a long position in a call and a put with different strike prices ($K_1$ for the call, and $K_2$ for the put, with $K_1 > K_2$), but with the same exercise date $T$ and on the same stock $S$.

List the payout on the exercise date, distinguishing three different areas for the $S_t(T)$ share price. When would an investor buy a strangle in the case $K_2 << K_1$ and $K_2 < S_0 < K_1$?

Z.O.Z. Remaining questions on the other side.
4. Suppose that $X(t)$ satisfies the following Stochastic Differential Equation (SDE):
\[ dX(t) = 0.02X(t)dt + \sigma X(t)dW(t), \]
and $Y(t)$ satisfies:
\[ dY(t) = \beta Y(t)dt + 0.1Y(t)dW(t). \]
Parameters $\beta, \sigma$ are positive constants and both processes are driven by the same Brownian Motion $W(t)$.
For a given process
\[ Z(t) = Y(t) + (\lambda - \sigma)t, \]
with $\lambda \in \mathbb{R}$.
(a). Find the SDE for $Z(t)$.
(b). For what values of $\beta, \sigma$ and $\lambda$ is process $Z(t)$ a martingale?

5. Let $\{(W_1(t), W_2(t)) : t \geq 0\}$ be a two-dimensional Brownian motion defined on a probability space $(\Omega, F, \mathbb{P})$. Consider the two processes $\{Z(t) : t \geq 0\}$ and $\{B(t) : t \geq 0\}$, defined by
\[ Z(t) = 1 + e^{-W_1(t)} \int_0^t e^{W_1(u)}dW_2(u), \]
and
\[ B(t) = \int_0^t \frac{1}{\sqrt{1 + Z^2(u)}}dW_1(u) - \int_0^t \frac{Z(u)}{\sqrt{1 + Z^2(u)}}dW_2(u). \]
(a). Use Lévy’s characterization to prove that process $\{B(t) : t \geq 0\}$ is a one-dimensional Brownian motion.
(b). Prove that the process $\{Z(t) : t \geq 0\}$ can be written as
\[ Z(t) = 1 + W_2(t) - \int_0^t (Z(u) - 1)dW_1(u) + \frac{1}{2} \int_0^t (Z(u) - 1)du. \]

6. Let $\{(W_1(t), W_2(t)) : t \geq 0\}$ be a 2-dimensional Brownian motion, defined on a probability space $(\Omega, F, \mathbb{P})$. Consider two price processes $\{S_1(t) : t \geq 0\}$ and $\{S_2(t) : t \geq 0\}$ with corresponding SDE given by:
\[ dS_1(t) = \alpha S_1(t)dW_1(t) + \beta S_1(t)dW_2(t) \]
\[ dS_2(t) = \gamma S_2(t)dt + \sigma S_2(t)dW_1(t), \]
where $\alpha, \beta, \gamma, \sigma$ are positive constants.
(a). Show that $\{S_1(t)S_2(t) : t \geq 0\}$ is a 2-dimensional Itô-process.
(b). Consider a finite expiration date $T$, and suppose the interest rate is constant, $R(t) = r$ for all $t > 0$. Show that the market price equations have a unique solution, and determine the risk-neutral probability measure $\tilde{\mathbb{P}}$ for the process $\{(S_1(t), S_2(t) : 0 \leq t \leq T)\}$. Please, make sure that your name is written down on each of the submitted solutions.