Retake exam

Topologie en Meetkunde, Block 3, 2021

Instructions

- The exam is 3 hours long (unless you are entitled to extra time).
- The exam is open book: you may use Hatcher or the lecture notes as a reference while working on it. However, you must sign and add to your exam the following declaration: "Hierbij verklaar ik dat ik de uitwerkingen van dit tentamen zelf heb gemaakt, zonder hulp van andere personen of van andere hulpmiddelen dan het cursusboek/dictaat/eigen aantekeningen".
- You have to justify all the claims you make. You may use results from the lectures or the book, but you should spell out what the hypothesis of the result are, and how they are satisfied.
- Try to be precise/clear: only one claim per sentence, only one key idea per paragraph.
- I recommend that you draw the spaces that appear in each of the exercises.

Some practical considerations:

- You must keep Teams open and your camera on for the duration of the exam. Once the time is up, you will have 15 minutes (still with the camera on) in order to submit the exam to me by email.
- You must submit a PDF. You may produce this PDF either using Latex or by scanning a handwritten document. If you do the later, use a high quality scanner. There are many phone apps that do this.
- I will be available through Microsoft Teams during the exam. You may send me a private message if you have any questions.
- After the exam is complete, store your physical copy in case it needs to be given to the University for archiving.
- I may arrange a little chat after the exam with some of you to go over what you wrote. You can simply leave when you finish, but keep an eye out in case I write to you on Teams.

Questions

Exercise 1 (1 point)**.** Let X be a topological space and let $A \subset X$ be a retract.

- Show that if X is contractible, then A is contractible too.
- Is the converse true?

Exercise 2 (1.5 points)**.** Let $\psi : \pi_1(\mathbb{S}^1, 1) \to \pi_1(\mathbb{S}^1, 1)$ be a group homomorphism. Show that there is a map $f : (\mathbb{S}^1, 1) \to (\mathbb{S}^1, 1)$ such that $f_* = \psi$.

Exercise 3 (1 point)**.** Find a pair of non-homeomorphic surfaces which are homotopy equivalent.

Exercise 4 (1.5 point)**.** Let $C = \mathbb{S}^1 \times \mathbb{R}$ denote the open cylinder. Find two non-homeomorphic compact surfaces A and B such that $A\#C$ and $B\#C$ are homotopy equivalent.

- Notation: Recall that $A\#C$ is obtained from A and C by removing an open disc from each of them and identifying the resulting boundary circles.

- Suggestion: Consider first simple examples of A in order to see what $A \# C$ looks like.

Exercise 5 (1.5 points)**.** Let A and B be two copies of $\mathbb{S}^2$, denote their north and south poles by N_A, S_A and N_B, S_B, respectively. Let X be the quotient of $A \coprod B$ where N_A is identified with N_B and S_A is identified with S_B.

- Endow X with a CW-structure. State explicitly how many cells of each dimension you use. Explain how they are attached (show this pictorially).
- Is X a surface?

Exercise 6 (1.5 point)**.** We continue with the same X as in the previous exercise. Compute the fundamental group of X, you can choose whatever basepoint $p \in X$ you prefer.

Exercise 7 (2 points)**.** We continue with the same X as in the previous exercise.

- Find a 2-sheeted, path-connected covering space $\tilde{X}$ of X.
- Describe the deck transformations of $\tilde{X}$. To which subgroup of $\pi_1(X, p)$ does $\tilde{X}$ correspond to?
- Find the universal cover of X. You must justify your answer by showing that the claimed space is simply-connected.