Utrecht University
Block 2, 2020-21

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Date: April 21, 2021

Name:________________ ID #:________ Signature:________

WISB272 Game Theory
Final Exam Resit

This exam contains 12 pages (including this cover page) and 6 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

The following rules apply:

- **Please, write your solutions in English!**
- **You have 3 hours to complete the exam**.
- **This is an open book exam.** You are allowed to use the textbook and/or lecture notes when working on it.
- **You are required to show your work on each problem on this exam.** All answers must be justified. A correct answer, unsupported by calculations and/or explanation will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might receive partial credit.
- **If you use a theorem or proposition from class or the notes or the textbook you must indicate this** and explain why the theorem can be applied.
- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- If you need more space, use the back of the pages; clearly indicate when you have done this.
- Do not write in the table to the right.

Problem	Points	Score
1	15	
2	10	
3	15	
4	15	
5	20	
6	20	
Total:	95	

Good luck!

1. Find the value of the game and a pair of optimal strategies for payers I and II for each of the following zero-sum games.

   (a) (5 points) Player II chooses a number $\alpha \in \{1, \sqrt{2}, -\sqrt{2}\}$, and player I tries to guess what number player II has chosen. If player I guesses the number correctly, then player I wins α^2 dollars from player II. Otherwise the payoff is 0.

(b) (10 points) A zero-sum game with the payoff matrix $A = \begin{pmatrix} 0 & -1 & 1 & -1 \\ -1 & 1 & 6 & -2 \\ -1 & 3 & 2 & -1 \\ -3 & 2 & -4 & -2 \\ -1 & -1 & 1 & 5 \end{pmatrix}$

2. (10 points) Consider the following general-sum game for two players:

	A	**B**	**C**
a	$(6,7)$	$(2,1)$	$(4,6)$
b	$(0,4)$	$(3,8)$	$(2,3)$
c	$(1,2)$	$(1,5)$	$(1,1)$

Find all Nash equilibria for this game.

3. Consider the following voting process. Three legislators are voting on whether to give themselves a pay raise. The raise is worth b but each legislator who votes for the raise incurs a cost of voter resentment equal to $c < b$. The outcome is decided by the majority vote. The legislators vote sequentially and publicly (that is, 2 sees 1's vote and 3 sees both 1 and 2's votes). (b) Find a Nash equilibrium for this game using backward induction. Show that it is best to go first. (c) Show that there are two Nash equilibrium in which 3 votes no. Clearly specify these Nash equilibria. Why cant these equilibria be found by backward induction? Is there something strange about these equilibria?

   (a) (10 points) Write this game in the extensive form. Find all subgame perfect equilibria. Is it best to vote first, second, or third?

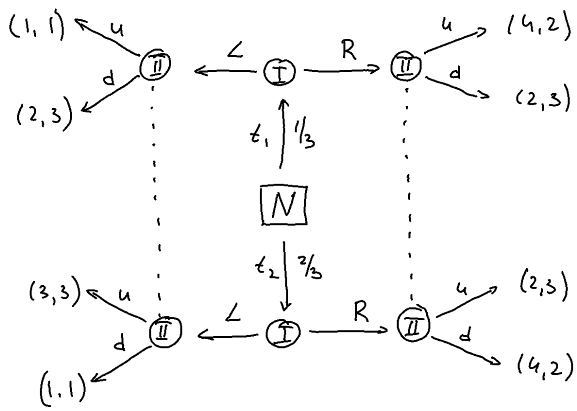
(b) (5 points) Show that there are two Nash equilibria for this game in which the third legislator votes against the pay raise. Are these Nash equilibria subgame perfect?

4. Consider a game with the following rules. Player I flips a coin and sees the result; player II does not see the result. Then player I makes a claim about the coin flip to player II, either claiming to have flipped Heads, or claiming to have flipped Tails. Player II can choose to Dispute the claim, or to Accept it. If player II chooses to Dispute, player I must show the coin. If player I lied, player II wins (with the payoffs -1 for player I and 1 for player II); if player 1 told the truth, player I wins (with the payoffs 1 for player I and -1 for player II). If player 2 chooses to Accept, then whatever player I claimed stands (regardless of what she actually flipped), and player II must flip the coin to compete with that claim. In this case, if player II flip outcome is the same as what player I claims to have (that is, both Heads or both Tails), it is a tie, and both have zero payoffs. If player II flip outcome is different from what player I claims to have, then the one who has Heads wins, with the payoff 1 for the winner and payoff -1 for the looser.

   (a) (5 points) Write this game in the extensive form of a Bayesian game. Mark information sets for each of the players.

(b) (10 points) Write the game in the normal form and identify all Nash equilibria.

5. Consider the following incomplete information game.



(a) (10 points) Find all separating perfect Bayesian equilibria in pure strategies.

(b) (10 points) Find all pooling perfect Bayesian equilibria in pure strategies.

6. Consider a symmetric evolutionary game given by the following bi-matrix

	A	**B**	**C**
A	$(0,0)$	$(1,2)$	$(1,3)$
B	$(2,1)$	$(0,0)$	$(2,3)$
C	$(3,1)$	$(3,2)$	$(0,0)$

(a) (10 points) Find all evolutionary stable strategies (ESS) for this game using an (equivalent) definition of ESS.

(b) (10 points) Find all ESS for this game using replicator dynamics.