Utrecht University
Block 2, 2020-21

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WISB272 Game Theory
Final

This is an open book exam. The following rules apply:

- You are **allowed to use the textbook and/or your notes** while working on it. **Collaborations on the final are not allowed.**
- You have 3 hours to work on the exam, plus 40 min to scan and submit it. The **deadline for submission is 15:10**. Students who are allowed extra time have additional 30 min, and deadline for exam submission for them is 15:40. **No late submission of the exam are possible.**
- Please write your solutions in **English**.
- Show your work on each problem. **All answers must be justified. Mysterious or unsupported answers will not receive credit**. A correct answer, unsupported by calculations and/or explanation will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- **If you use a theorem or proposition from class or the notes or the textbook you must indicate this** and explain why the theorem may be applied.
- **Organize your work**, in a reasonably neat and coherent way. Work scattered all over the page without a clear ordering will receive very little credit.
- You are required to **submit your final as a single pdf-file to Blackboard ('final exam' assignment)**.
- **The pdf file with your solutions should be formatted in the following way:**
  - The first page should include your name (printed) followed by your university ID (also printed) on the top of the page. Below, please include the following statement, followed by your signature and date:
    "Hereby I declare that I have produced the solutions to the exam questions by myself, without assistance of other people or of other means except for the textbook and lecture notes."
    The first page should not contain anything else (no solutions on the first page).
  - You should have a separate page for each subproblem in the final, stating the number of the subproblem in the left upper corner (e.g. 1a), 1b), etc). If you choose not to do a particular subproblem, please, still include an empty page (with this subproblem number in the left upper corner). Also make sure that you have all the subproblems in the right order!

Please, follow this instructions precisely, as failing to do so may result in loosing parts of your solutions!

Good luck!

1. Find the value of the game and a pair of optimal strategies for payers I and II for each of the following zero-sum games, given by their payoff matrices.

   (a) (5 points) $A = \begin{pmatrix} 5 & 4 & 1 & 0 \\ 4 & 3 & 2 & -1 \\ 0 & -1 & 4 & 3 \\ 1 & -2 & 1 & 2 \end{pmatrix}$

   (b) (10 points) $A = \begin{pmatrix} d_1 & 0 & 0 \\ 0 & d_2 & 0 \\ 0 & 0 & d_3 \end{pmatrix}$, where $d_1, d_2, d_3 > 0$. Does this game have a pair of optimal strategies that are not fully mixed (for at least one of the players)? Explain.

2. (10 points) Consider the following general-sum game for two players:

	A	**B**	**C**
a	$(-1, 1)$	$(0, 2)$	$(0, 2)$
b	$(2, 1)$	$(1, -1)$	$(0, 0)$
c	$(0, 0)$	$(1, 1)$	$(1, 2)$

   Find all Nash equilibria for this game.

3. Consider the following game. Player I has two chips marked I. She can place them in two identical open boxes in any way she wants (player II can observe the result of player I's move). Player II has one chip marked II. After player I is done with her move, player II chooses a box to put her chip in. To decide the payoffs, the referee picks one of the boxes at random (each box is picked with probability $\frac{1}{2}$). Then the payoff is decided as follows

   - If the selected box is empty, each player receives \$6.
   - If the box is not empty, the referee selects a chip from the box uniformly at random. The player whose chip is selected receives \$12, and the other player does not get anything.

   (a) (10 points) Write this game in the extensive form and find the subgame perfect equilibrium.

   (b) (5 points) Write the game in the normal form. Are there any other Nash equilibria for this game? Explain your answer.

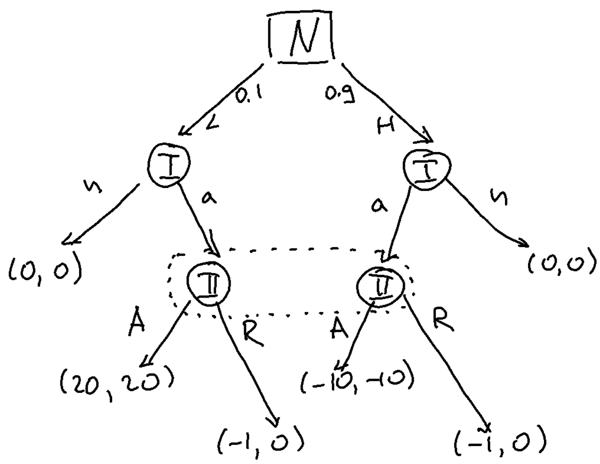
4. Consider the following zero-sum game. Player II chooses a number a in the set $\{1, 2, 3\}$. Then one of the numbers not chosen is selected uniformly at random and shown to player I. Then player I tries to guess which number player II chose. If player I guesses the number a right, then player II pays a to player I. Otherwise, the payoff of both players is 0.

   (a) (10 points) Using moves by nature, write this game in the extensive form. Mark information sets and describe behavioral strategies for each of the players.

   (b) (5 points) Write the game in the normal form.

5. Consider the following game between a possible applicant to a Ph.D. program in Mathematics and the Admission Committee. Admission Committee believes that the student hates mathematics ('H') with probability 0.9 and loves mathematics ('L') with probability 0.1. The student decides whether or not to apply to the Ph.D. program. If the student does not apply ('n'), both the student and the committee get payoff 0. If student applies ('a'), then the committee is to decide whether to accept ('A') or reject ('R') the student. If the committee rejects, then

committee gets payoff 0, and student gets payoff -1. If the committee accepts the student, the payoffs depend on whether the student loves or hates mathematics. If the student loves mathematics, she will be successful and the payoffs will be 20 for each player. If she hates mathematics, the payoffs for both the committee and the student will be -10.



(a) (10 points) Find all separating perfect Bayesian equilibria in pure strategies.

(b) (10 points) Find all pooling perfect Bayesian equilibria in pure strategies.

6. Consider a symmetric evolutionary game given by the following bi-matrix

	A	**B**
A	$(2, 2)$	$(4, 5)$
B	$(5, 4)$	$(1, 1)$

(a) (10 points) Find all evolutionary stable strategies (ESS) for this game using an (equivalent) definition of ESS.

(b) (10 points) Find all ESS for this game using replicator dynamics.