# Retake Exam – Elementaire Getaltheorie

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No calculators or other electronic devices are allowed.

**Problem 1** (24 points). *Decide (with complete reasoning!) for the following assertions whether they are true or not.*

*(a) If $d$ is odd, $d^2 - 1$ is divisible by* 8.

*(b) The number* 12317327 *is prime.*

*(c) There is an integer $n$ such that $n^2 - 5$ is divisible by* 41.

*(d) If $a^2 + b^2 = c^2$, then $abc$ is divisible by* 5.

*(e) There is an integer $n$ such that $n \equiv 4 \mod 48$ and $n \equiv 5 \mod 27$.*

*(f) There exists an irrational number whose square and cube are rational.*

**Problem 2** (6 points). *Give two pairs of integers $(x, y)$ such that $23x + 17y = 2$.*

**Problem 3** (9 points). *Let $n > 4$ be an integer. Show that $(n-1)!$ is divisible by $n$ if and only if $n$ is not prime.*

**Problem 4** (12 points). *(a) Give the continued fraction expansion of $\sqrt{10}$.*

*(b) Find two pairs of nonequal integers $(x, y)$ such that $x^2 - 10y^2 - 20y = 11$.*

**Problem 5** (12 points). *Give concrete examples of*

*(a) a number such that its continued fraction expansion is purely periodic (zuiver periodiek);*

*(b) a primitive root modulo* 11*;*

*(c) an integer $n > 100$ such that there exist no integers $x, y$ with $x^2 + y^2 = n$.*

**Problem 6** (12 points). *Find all primes $p$ such that the following is true: there is an integer $k > 2$ such that $2^{k-2} + 3^{k-2} + 6^{k-2} - 1$ is divisible by $p$.*