

Third exam – Elementaire Getaltheorie

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Theorems from class or book can be used when marked as such. Exercises from the exercise sessions (werkcolleges) cannot be used without repeating their solution.

Problem 1 (4 points). Write $\frac{1153}{140}$ as a continued fraction.

Problem 2 (8 points). Determine for $n = 1236$ and $n = 1153$ whether they are the sum of two squares. If yes, find one pair of integers (x, y) with $n = x^2 + y^2$. (Hint: You may use that $140^2 + 1$ is divisible by 1153.)

Problem 3 (8 points). Show that for every natural number $a > 2$, there is are natural numbers b, c with $a^2 + b^2 = c^2$. (Hint: Distinguish the cases a even and a odd.)

Problem 4 (8 points). Let n be a positive integer of the form $8k + 7$ with $k \in \mathbb{Z}$. Show that n is of the form $x^2 + y^2 + z^2 + w^2$ with x, y, z and w **positive** integers.