Third exam – Elementaire Getaltheorie

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Theorems from class or book can be used when marked as such. Exercises from the exercise sessions (werkcolleges) cannot be used without repeating their solution.

Problem 1 (4 points). Write \( \frac{1153}{140} \) as a continued fraction.

Problem 2 (8 points). Determine for \( n = 1236 \) and \( n = 1153 \) whether they are the sum of two squares. If yes, find one pair of integers \((x, y)\) with \( n = x^2 + y^2 \). (Hint: You may use that \( 140^2 + 1 \) is divisible by 1153.)

Problem 3 (8 points). Show that for every natural number \( a > 2 \), there is are natural numbers \( b, c \) with \( a^2 + b^2 = c^2 \). (Hint: Distinguish the cases \( a \) even and \( a \) odd.)

Problem 4 (8 points). Let \( n \) be a positive integer of the form \( 8k + 7 \) with \( k \in \mathbb{Z} \). Show that \( n \) is of the form \( x^2 + y^2 + z^2 + w^2 \) with \( x, y, z \) and \( w \) positive integers.