First exam – Elementaire Getaltheorie

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In all problems write your solution in detail. Each step has to be proven or cited from class.

**Problem 1** (10 points). For the following two equations, decide whether they have solutions with $x, y \in \mathbb{Z}$. If yes, give two different pairs $(x, y)$ of solutions.

(a) $447x + 408y = -3$

(b) $447x + 408y = 7$

Decide furthermore if the system of congruences

$$a \equiv -3 \mod 447$$
$$a \equiv 7 \mod 408$$

has a solution $a \in \mathbb{Z}$ and if yes, give such a solution.

**Problem 2** (10 points). Let $a$ be an arbitrary integer.

(a) Compute the remainder of $a^{36}$ if we divide by 36.

(b) Show that $a^{36} - 1$ is not a prime number.

**Problem 3** (10 points). Recall that the sum of positive divisors $\sigma(n)$ of a natural number $n$ with prime factorization $p_1^{k_1} \cdots p_r^{k_r}$ with $p_1 < \cdots < p_r$ equals

$$\prod_{i=1}^r \frac{p_i^{k_i+1} - 1}{p_i - 1}.$$ 

Give a similar formula for

$$\sum_{0 < d | n} d^2.$$