Retake Final WISB372, 21-12-2011, 14-17

Problem 1 [40 pts.] Consider the following optimal control problem: maximize $\int_0^T (x(t) - ax^2(t) - u^2(t))dt$ over all control functions $u : [0,T] \to \mathbb{R}$ such that $\dot{x} = u - \delta x$ and $x(0) = x_0$. Here $a \geq 0$, $\delta \geq 0$ are parameters and x_0 is a given initial state.

a. Use the Pontryagin MP to determine the candidate-optimal control functions and their associated trajectories for the special case $a = 0$, $\delta = 0$.

b. Use the Pontryagin MP to determine the candidate-optimal control functions and their associated trajectories for the case $a \geq 0$, $\delta \geq 0$.

Problem 2 [25 pts.] You are the only player in a game with N rounds that involves an urn, filled with balls. Your purpose is to maximize your expected total winnings from this game.

Initially, the urn is filled with B black balls and $R > N$ red balls. In each round you may decide either to draw a ball – at random – from the urn or not to draw. If you don't draw, nothing happens to you financially. If you choose to draw, you must first pay a dollar; thereupon, if you draw a red ball, you win $W > 1$ dollars and if you draw a black ball you get zero dollars.

The game has a remarkable rule: red balls, when drawn, may not be returned to the urn. However, black balls, when drawn, must be returned to the urn (so there are always B black balls in the urn, whereas the number of red balls can only go down).

a. Formulate the above game as a DP-model in standard form.

b. Determine the optimal policy for this model by means of the DPA.

c. Intuitively, your answer in part b should make good sense. How? To check it for correctness, test your answer in part b in the very special case $B = 0$.

Problem 3 [35 pts.] Consider the following problem: minimize $\int_0^T \dot{x}^2(t)dt$ over all $T > 0$ and all continuously differentiable functions $x : [0,T] \to \mathbb{R}$ such that $x(0) = 0$ and $x(T) + T + 1 = 0$.

a. Reformulate this problem as a standard optimal control problem with free terminal time by considering $z(t) := x(t) + t$ and $u(t) := \dot{z}(t)$.

b. Find the candidate-optimal control functions for the reformulated problem in part a.

c. Use the outcome in part b to find the candidate-optimal solutions for the original problem in part a. *Hint:* there are two such functions and you must determine both.