Tentamen Speltheorie (WISB272)
June 2023

- This is a **closed book exam**, and you are **not allowed to use a cheat sheet**.
- Put your ID card on the table. Turn off your cell phone and put it in your bag.
- You have 3 hours for the exam (plus an additional 30 minutes if you have extra time).
- The exam consists of five questions. The points you can earn for each (sub)question is indicated under each question. The total number of points you can earn is 100.
- Please **use a new sheet of paper for each question** and **write your name and student number on each sheet**. This will help avoid delays with grading.
- Your solutions can be in **English or in Dutch**. Try to be consistent in your choice of language so as to minimize the risk of confusion.
- Show your work on each problem. **If you are asked to prove a result, provide a formal proof**.
- **If you use a theorem or proposition from the lectures or lecture notes, clearly indicate this**. You do not need to name the result or provide its number; but don't forget to verify that the conditions of the statements you use have been met.
- **Good luck!**

Question 1 (new sheet of paper)

Prove that if an action is strictly dominated, then it is never a best response, i.e., it is not a best response to any conjecture. [10 points]

Question 2 (new sheet of paper)

There are two players. The players are roommates who each need to choose how much time to use to clean their apartment. That is, each player $i \in \{1, 2\}$ can choose an amount of time $t_i \geq 0$ to clean. If their choices are t_1, t_2, then player i's payoff is given by $u_i(t_i, t_{-i}) = (10 - t_{-i})t_i - t_i^2$. (Thus, the players dislike cleaning, and the more one roommate cleans, the less attractive for the other roommate it is to clean.)

(a) What is the best response correspondence of each player i? [5 points]

(b) Which actions are rationalizable? [20 points]

You can **restrict attention to pure strategies** throughout.

Question 3 (new sheet of paper)

Consider the following game:

	ℓ	c	r
T	1,1	2,-2	-2,2
B	1,1	-2,2	2,-2

(a) Show that the game has no Nash equilibrium in which player 1 chooses a pure strategy. [5 points]

(b) Find all Nash equilibria of the game. [15 points]

Question 4 (new sheet of paper)

Find the value and optimal strategies of the following zero-sum game [20 points]:

$$\begin{pmatrix} 1 & -4 & 5 & 4 \\ 0 & 0 & 2 & 1 \\ 4 & 6 & 1 & 0 \end{pmatrix}$$

Question 5 (new sheet of paper)

Ann has two envelopes. She puts 10^n euro in one envelope, and 10^{n+1} euro in the other envelope, where the number n is chosen with equal probability from $\{1, 2, 3, 4, 5\}$. (For example, the probability that $n = 2$ is $\frac{1}{5}$.) She randomly hands one envelope to Bob and the other to Carol. (So, conditional on n, the probability that Bob has the envelope with 10^n euro is $\frac{1}{2}$, and likewise for Carol.) Bob and Carol are seated in different rooms and cannot communicate. Everyone knows how Ann has selected the amounts of money in each envelope and how the envelopes have been distributed among Bob and Carol.

(a) Write down the information structure, taking a state to be a pair (m, m') such that, if the state is (m, m'), the amount of money in Bob's envelope is 10^m while the amount of money in Carol's envelope is $10^{m'}$. [5 points]

(b) Show that for any state $\omega = (m, m')$, Bob's conditional expectation of the amount of money in Carol's envelope (given his information) strictly exceeds the amount of money in his own envelope if and only if $m \leq 5$. Likewise, for any state $\omega = (m, m')$, Carol's conditional expectation of the amount of money in Bob's envelope (given her information) strictly exceeds the amount of money in her own envelope if and only if $m' \leq 5$. [5 points]

Bob looks inside his envelope and finds that it contains 10^4 euros; Carol looks inside her envelope and finds that it contains 10^5 euros.

(c) Ann privately asks Bob and Carol whether they would be willing to switch envelopes; she then tells each of them what the other answered and repeats the question. Assuming that a player is willing to switch if and only if he/she expects the other's envelope to contain strictly more money than his/her own envelope, do players say "yes" or "no" when asked for the first time whether they are willing to switch? What about the second time? [15 points]