Re-sit (hertentamen) Speltheorie (WISB272)
July 2023

- This is a **closed book exam**, and you are **<u>not</u> allowed to use a cheat sheet**.
- Put your ID card on the table. Turn off your cell phone and put it in your bag.
- You have 3 hours for the exam (plus an additional 30 minutes if you have extra time).
- The exam consists of five questions. The points you can earn for each (sub)question is indicated under each question. The total number of points you can earn is 100.
- Please **use a new sheet of paper for each question** and **write your name and student number on each sheet**. This will help avoid delays with grading.
- Your solutions can be in **English or in Dutch**. Try to be consistent in your choice of language so as to minimize the risk of confusion.
- Show your work on each problem. **If you are asked to prove a result, provide a formal proof**.
- **If you use a theorem or proposition from the lectures or lecture notes, clearly indicate this**. You do not need to name the result or provide its number; but don't forget to verify that the conditions of the statements you use have been met.
- **Good luck!**

Question 1 (new sheet of paper)

Let $G = (N, (A_i)_{i\in N}, (u_i)_{i\in N})$ be a finite strategic-form game. For $i \in N$, let $Z_i \subseteq A_i$ be a subset of actions. Suppose that for each player $i \in N$ and $z_i \in Z_i$, there is $z_{-i} \in Z_{-i}$ such that z_i is a best response to z_{-i}, i.e.,

$$\forall s_i \in \Delta(A_i): \qquad u_i(z_i, z_{-i}) \geq \sum_{a_i\in A_i} s_i(a_i)u_i(a_i, z_{-i}).$$

Prove, without using propositions or lemmas from the lecture notes, that every action $z_i \in Z_i$ is rationalizable. [10 points]

Question 2 (new sheet of paper)

Consider the following game: Two players, Ann and Bob, pick a number from $\{0, 1, \ldots, 100\}$. If Ann picks a number $a \in \{0, 1, \ldots, 100\}$ and Bob picks a number $b \in \{0, 1, \ldots, 100\}$, her payoff is

$$u_{\text{Ann}}(a, b) = \begin{cases} 2 - (a-b)^2 & \text{if } a < b; \\ -(a-b)^2 & \text{otherwise.} \end{cases}$$

Similarly, if Ann picks a number $a \in \{0, 1, \ldots, 100\}$ and Bob picks a number $b \in \{0, 1, \ldots, 100\}$, Bob's payoff is

$$u_{\text{Bob}}(b, a) = \begin{cases} 2 - (a-b)^2 & \text{if } b < a; \\ -(a-b)^2 & \text{otherwise.} \end{cases}$$

(a) Show that for each player, any pure strategy (action) $c \in \{0, 1, \ldots, 99\}$ is a best response to some conjecture. [5 points]

(b) Determine the rationalizable strategies for both players. [20 points]

Question 3 (new sheet of paper)

Consider the following game

	ℓ	c	r
T	5,5	3,0	0,2
M	5,1	2,1	1,0
B	0,0	2,5	4,2

(a) Find all rationalizable strategies. [5 points]

(b) Find all Nash equilibria. [15 points]

Question 4 (new sheet of paper)

Consider the zero-sum game with payoff matrix

$$\mathcal{A} = \begin{pmatrix} 2 & 0 & 1 & 4 \\ 1 & 2 & 5 & 3 \\ 4 & 1 & 3 & 2 \end{pmatrix}$$

Find the value and determine all optimal strategies for the players. [20 points]

Question 5 (new sheet of paper)

There are two players, labeled 1 and 2. The information structure is $(\Omega, (\Pi_i)_{i\in\{1,2\}}, (\mathbb{P}_i)_{i\in\{1,2\}})$, where $\Omega = \{\omega_1, \omega_2, \omega_3, \omega_4, \omega_5, \omega_6, \omega_7\}$, $\mathbb{P}_1 = \mathbb{P}_2 =: \mathbb{P}$ with

$$\begin{aligned} &\mathbb{P}(\{\omega_1\}) = 4/32; && \mathbb{P}(\{\omega_5\} = 7/32; \\ &\mathbb{P}(\{\omega_2\}) = 2/32; && \mathbb{P}(\{\omega_6\} = 2/32; \\ &\mathbb{P}(\{\omega_3\}) = 8/32; && \mathbb{P}(\{\omega_7\} = 4/32; \\ &\mathbb{P}(\{\omega_4\}) = 5/32; && \end{aligned}$$

and

$$\begin{aligned} \Pi_1 =& \big\{\{\omega_1, \omega_2, \omega_3\}, \{\omega_4, \omega_5, \omega_6\}, \{\omega_7\}\big\}; \\ \Pi_2 =& \big\{\{\omega_1, \omega_2, \omega_4\}, \{\omega_3, \omega_5, \omega_6, \omega_7\}\big\}. \end{aligned}$$

Consider the event $E := \{\omega_1, \omega_3, \omega_4, \omega_5\}$, and suppose that the true state is ω_6.

(a) Calculate the (conditional) probability that each player assigns to E. [5 points]

(b) Assume that players repeatedly announce the conditional probability they assign to E, in the following way: In each round $k \in \{1, 3, 5, \dots\}$, player 1 announces the conditional probability that she assigns to E (given her information in round k); and in each round $k \in \{2, 4, 6, \dots\}$, player 2 announces the conditional probability he assigns to E (given his information in round k). Will players' conditional probabilities converge? If so, to what value do they converge, and in which round? Support your answer by specifying how the players' conditional probabilities change from one round to the next. [20 points]