1). $3x^2(x^3-x^2-5x-3)$ $\quad$ $x=-1$ voldoet: $-1-1+5-3=0$

$$x+1 \,/\, x^3-x^2-5x-3 \,\backslash\, x^2-2x-3$$
$$\underline{x^3+x^2}$$
$$-2x^2-5x$$
$$\underline{-2x^2-2x}$$
$$-3x-3$$

$x=-1$ voldoet weer: $1+2-3=0$

$x^2-2x-3=(x+1)(x-3)$

Dus de factoristatie is: $3x^2(x+1)^2(x-3)$

2). Willen: $f(x)f'(x)=-1$ met $f(x)=\sqrt{a-4x^2}$

dus $f'(x)=\dfrac{-4x}{\sqrt{a-4x^2}}$

Er volgt: $f(x)f'(x)=-4x$ mits $a-4x^2>0$.

De eis is dus dat $x=+\frac{1}{4}$, en deze voldoet indien $a>4x^2=\frac{1}{4}$. Dus indien $a>\frac{1}{4}$ is er precies 1 opl, nl $x=\frac{1}{4}$.

3). Dit is een verkapte vorm van std-lim $\lim\limits_{x\to\infty}\dfrac{x^t}{e^x}=0$. (Hst 3)

Om dit te gebruiken, schrijven we

$$\frac{x^{2022\pi}}{e^{3x-61}}=e^{61}\left(\frac{x^{674\pi}}{e^x}\right)^3$$

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Nu de std lim gebruiken met $t=674\pi$: $\lim\limits_{x\to\infty}\dfrac{x^{674\pi}}{e^x}=0$.

Dan is $\lim\dfrac{x^{2022\pi}}{e^{3x-61}}=e^{61}(0)^3=0$. (rekenregels voor lim, hst 2)

4). $f(x)=\arcsin\dfrac{x-1}{x}$;

$$\left(\frac{x-1}{x}\right)'=\left(1-\frac{1}{x}\right)'=\frac{1}{x^2}$$

$$f'(x)=\frac{1}{x^2\sqrt{1-\left(\frac{x-1}{x}\right)^2}}=\frac{1}{x\sqrt{x^2-(x-1)^2}}=\frac{1}{x\sqrt{2x-1}}$$

Nu $x=5$, geeft $f'(5)=\dfrac{1}{5\sqrt{10-1}}=\dfrac{1}{15}$.

5). $f(x)=\sqrt{1+x}$ $\qquad f(0)=1$

$f'(x)=\dfrac{1}{2\sqrt{1+x}}=\frac{1}{2}(1+x)^{-1/2}$ $\qquad f'(0)=\frac{1}{2}$

$f''(x)=-\frac{1}{4}(1+x)^{-3/2}$ $\qquad f''(0)=-\frac{1}{4}$

$f'''(x)=\frac{3}{8}(1+x)^{-5/2}$ $\qquad f'''(0)=\frac{3}{8}$

$$T_3(x)=f(0)+f'(0)x+\frac{f''(0)}{2}x^2+\frac{f'''(0)}{6}x^3$$

$$=1+\frac{1}{2}x-\frac{1}{8}x^2+\frac{1}{16}x^3$$

We willen hiermee $\sqrt{2}$ benaderen, daarom kiezen we $x=1$:

$$\sqrt{2}\approx 1+\frac{1}{2}-\frac{1}{8}+\frac{1}{16}=\frac{16+8-2+1}{16}=\frac{23}{16}$$

6). $x^2+x-6=(x+3)(x-2)$; we gaan breuksplitsen!

$$\frac{5x}{(x+3)(x-2)}=\frac{A}{x+3}+\frac{B}{x-2}$$ zdd: $(x-2)A+(x+3)B=5x$

$$\begin{cases}A+B=5\\ 3B-2A=0\end{cases}\qquad A=3,\ B=2.$$

dus: $$\int\frac{5x}{x^2+x-6}dx=\int\frac{3}{x+3}dx+\int\frac{2}{x-2}dx$$

$$=3\log|x+3|+2\log|x-2|+c.$$

7). Subs. $u=\sqrt{1+x}$, ~~dus~~ $du=\dfrac{dx}{2\sqrt{1+x}}$ of $2u\,du=dx$.

x	u
2	$\sqrt{3}$
-1	0

$$\int_0^{\sqrt{3}}\frac{2u\,du}{1+u}=\int_0^{\sqrt{3}}2-\frac{2}{1+u}\,du$$

$$=\underbrace{2u-2\log|1+u|}\Big|_0^{\sqrt{3}}=\boxed{2\sqrt{3}-2\log(1+\sqrt{3})}$$ Hm?!

check: diff geeft $2-\dfrac{2}{1+u}=\dfrac{2u}{1+u}$

OK ook met terugsubs dan voor de check:

$2u-2\log|1+u|=2\sqrt{1+x}-2\log|1+\sqrt{1+x}|$,

met afgeleide $\dfrac{1}{\sqrt{1+x}}-\dfrac{2}{1+\sqrt{1+x}}\cdot\dfrac{1}{2\sqrt{1+x}}=\dfrac{1}{\sqrt{1+x}}\left(1-\dfrac{1}{1+\sqrt{1+x}}\right)$

$$=\frac{1}{1+\sqrt{1+x}}.$$

8) $\begin{cases} x^2y' = 2xy - 3 \\ y(-1) = 1 \end{cases}$ Inhom. beginwaardeprobleem

Eerst hom: $x^2y' = 2xy$ ~~f~~

Scheiden: $\frac{dy}{y} = 2\frac{dx}{x}$, $\log|y| = 2\log|x|$

$y = cx^2$

~~(Je kunt dit ook zien door op te merken dat~~

~~$d(x^2y) = 2xy\,dx + x^2dy$~~).

Nu inhom door var. van const:

$y = cx^2$

$y' = c'x^2 + 2cx$ $\Big|$ $x^2y' - 2xy = c'x^4 + \cancel{2cx^3} - 2\cancel{cx^3} \underset{\text{eis}}{\overset{\uparrow}{=}} -3$

Hieruit leren we $c' = \frac{-3}{x^4}$, dus $c = x^{-3} + c$

De alg. opl. is dus $y = (x^{-3}+c)x^2 = \frac{1}{x} + cx^2$.

De opl. van beginwaardeprobleem vinden we door beginwaarde in te vullen: $x=-1$, $y=1$; dwz $1 = -1 + c$,

dus $c = 2$ en opl. $y = \frac{1}{x} + 2x^2$.

9). We beginnen met de somformules in te zetten:

$\sin(\frac{\pi}{3} - x) = \sin\frac{\pi}{3}\cos x - \cos\frac{\pi}{3}\sin x$

$\sin(\frac{\pi}{3} + x) = \sin\frac{\pi}{3}\cos x + \cos\frac{\pi}{3}\sin x$ $\times$

$\sin^2\frac{\pi}{3}\cos^2 x - \cos^2\frac{\pi}{3}\sin^2 x = \frac{3}{4}\cos^2 x - \frac{1}{4}\sin^2 x.$

$= \frac{1}{2}\cos^2 x + \frac{1}{4}(\cos^2 x - \sin^2 x)$

Dan nog factor $\sin x$ erbij:

$\sin x \sin(\frac{\pi}{3} - x)\sin(\frac{\pi}{3} + x)$

$= \frac{1}{2}\sin x\cos^2 x + \frac{1}{4}\sin x(\cos^2 x - \sin^2 x)$

$= \frac{1}{4}\sin 2x\cos x + \frac{1}{4}\cos 2x\sin x = \frac{1}{4}\sin(3x)$

Dit gebruiken we met $x = 10°$, zodat $3x = 30° = \frac{\pi}{6}$.

Dus $\sin 10° \sin 50° \sin 70° = \frac{1}{4}\sin 30° = \frac{1}{8}$.