Utrecht University
Utrecht University School of Economics

Retake exam Econometric Methods (ECRMECM)

Thursday, 5 February 2021, 9:00-11:00 CET

- Please do not post copies on the Internet.

Remarks:
- This entrance test consists of 4 questions (10 sub-questions) on 5 numbered pages (included front page).
- **This exam is an open book exam in which you are allowed to make use of the lecture notes.**
- Write your name and registration number on each page of your exam.
- You should write down your answers in full, as if it were an on campus exam.
- Please make pictures of all pages of your answers (with a camera, copy the pictures in **ONE Word document**, and send it to W.H.J.Hassink@uu.nl.
- Please do not post copies of this exam on the Internet.

By taking the endterm exam of Econometric Methods ECRMECM of February 5th 2021, you automatically confirm that

“*By taking this exam I declare that I will formulate the answers myself, without the help of others, and without using unauthorized tools, and take the exam according to its instructions. Violation of these rules is regarded as fraud / plagiarism*."

Note that this exam is an open book exam.

Questions

In the questions below – unless otherwise stated – we consider the linear regression equation

$$y_i = \boldsymbol{\beta}'\mathbf{x}_i + u_i \qquad i=1,\ldots,n$$

for which y is the dependent variable. $\boldsymbol{\beta}$ is a (k+1) dimensional column vector. $\mathbf{x}$ is the (k+1) dimensional column vector of explanatory variables, and u is an error term. Subscript i refers to the i-th individual. n is the number of observations of the data set.

Question 1 (Chapter 7, 8)

a) The material of Chapter 7 gives the expected value of a random variable V, that follows a Chi square distribution with n degrees of freedom. Thus $V \sim \chi^2_{(n)}$

Question: If $V = (n-k-1)\cdot\dfrac{\hat{\sigma}_u^2}{\sigma_u^2}$, please calculate $E(V)$ and $E(\hat{\sigma}_u^2)$

b) For the $\hat{\boldsymbol{\beta}}$, we know that $E(\hat{\boldsymbol{\beta}} \mid \mathbf{X}) = \boldsymbol{\beta}$, $Var(\hat{\boldsymbol{\beta}} \mid \mathbf{X}) = \sigma_u^2(\mathbf{X}'\mathbf{X})^{-1})$, $E(\mathbf{u} \mid \mathbf{X}) = \mathbf{0}$ and $Var(\mathbf{u} \mid \mathbf{X}) = \sigma_u^2\mathbf{I}_n$, $\mathbf{u} \mid \mathbf{X}$ follows a multivariate Normal distribution.

Question: please derive the test statistic for

$H_0 : \mathbf{R}\boldsymbol{\beta} = \mathbf{r}$

R: q x (k+1) matrix, which can be used for testing.
$\mathbf{r}$ is a q-dimensional vector (q is the number of restrictions)

$H_1 : H_0$ not true

Please motivate your answer.

c) For

$$\begin{pmatrix} \hat{\beta}_0 \\ \hat{\beta}_1 \\ \hat{\beta}_2 \end{pmatrix} \mid \mathbf{X} \sim Normal(\begin{pmatrix} 8 \\ 2 \\ -1 \end{pmatrix}, \begin{pmatrix} 9 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 2 \end{pmatrix})$$

Question: please formulate the corresponding Chi-square statistic.

Question 2 (Chapter 9, 10)

a) Is it necessary to assume Normality of the error term u to calculate the robust standard errors of the estimated OLS estimator? Please motivate your answer.

Question 3 (Chapters 11, 12)

a) Consider the second-order moving average model (the MA(2) model)

$$u_t = e_t + \alpha_1 e_{t-1} + \alpha_2 e_{t-2} \qquad t = 3,\ldots,T$$

for which the error term e_t is i.i.d. (identically and independently distributed), with expected value zero and constant variance: $Ee_t = 0$ and $Var(e_t) = \sigma_e^2$.

Question: Could you please derive the $(T-2)\text{x}(T-2)$ covariance matrix of the error terms?

b) Is the second-order moving average model (the MA(2) model) a covariance stationary model? Please explain.

c) Under which assumptions can we obtain unbiased parameter estimates of the error-correction model

$$\Delta y_t = \alpha_0 + \alpha_1 \Delta y_{t-1} + \gamma_0 \Delta x_t + \gamma_1 \Delta x_{t-1} + \delta(y_{t-1} - \beta x_{t-1}) + \varepsilon_t$$

Question 4 (Chapter 13)

a) For the linear regression equation with one explanatory variable

$$y_{it} = \alpha_i + \beta_1 x_{it} + u_{it} \qquad i=1,\dots,n;\ t=1,\dots,T$$

Show that the within estimator and the first-differences estimator of β_1 are equal for T=2.

Next, the following panel data model is estimated.

(1) $$wage_{it} = \beta_0 + \beta_1 hours_{it} + \beta_2 tenure_{it} + \gamma t + v_{it}\ , \qquad i=1,...,N; t=1,...,T$$

$$v_{it} = \alpha_i + u_{it}$$

where $wage_{it}$ is the logarithm of the wage rate and $hours_{it}$ is the number of working hours per week and $tenure_{it}$ is the years of working experience. The individual is indexed by *i* and the time by *t*. The individual-specific effect (denoted by α_i) and the idiosyncratic error term (u_{it}) are assumed to be independent of $hours_{it}$ and $tenure_{it}$. Moreover, $hours_{it}$ and $tenure_{it}$ are strictly exogenous.

b) The parameters of equation (1) are estimated using a pooled OLS estimator. Please show that v_{it} and v_{it-1} are correlated.

Next, we allow the random effect a_i of equation (1) to be correlated with *hours* and/or *tenure* and/or time and for this reason the model is estimated using a first difference estimator.

c) How can the γ of equation (1) be identified from the first-difference (FD) estimator?

< End of the exam >