Utrecht University | School of Economics

Course name: Econometrics (Wisb377)
Date examination: November 7, 2019
Duration 2.5 hours; from <10:00> to <12:30>
Examination: Endterm
Total number of pages: 4
Total number of exercises: 6

Full name :---

Student ID Number : --

Copy your ID Number on every separate sheet/answering paper if required. DON'T PUT YOUR NAME ON THE ANSWERING PAPER!

Signature : ---

- Put your ID on your table for inspection.

Exam instructions

At the start of the exam

- Candidates who arrive 30 minutes after the time scheduled for the start of the examination will not be permitted entry to the examination room.

During the examination

- Nobody is allowed to leave the room within the first 30 minutes after the start of the exam.
- You are not allowed to go to the restroom unless you have permission of the Examiner or an invigilator.
- **MOBILE PHONES AND OTHER COMMUNICATION DEVICES ARE ONLY ALLOWED WHEN SWITCHED OFF AND STORED IN CLOSED BAGS**.
- *It is a closed book exam. It is* ***not*** *allowed to use any study aids such as books, readers, (preprogrammed) calculators*
- You may use a simple calculator and a dictionary (without any [handwritten] notes in it).
- The exam form is **NOT** allowed to be taken home by the candidate

Results/Post-examination regulations:

- The results of the examination will be announced on Blackboard within two weeks of the exam date. At the same time the time & place of the exam inspection will be announced.
- We do not discuss exam results over the phone or by email.
- After the announcement of the exam results in OSIRIS you have four weeks within which to lodge an appeal against your grade.
- Four weeks after the results of this exam are published, the original exam is available to you, when a declaration is signed, stating that no appeal has been made or will be made.
- You can request a photocopy of your answers at the Student Desk up and until four weeks after publication of the results.

Question 1)

For the OLS estimator $\hat{\boldsymbol{\beta}} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{y}$ of the linear regression equation $\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{u}$, for which $\mathbf{y}$ and $\mathbf{u}$ are n-dimensional vectors, $\boldsymbol{\beta}$ is a (k+1)-dimensional vector , $\mathbf{X}$ is a (n x (k + 1)) dimensional matrix,

$$\mathbf{u} \,|\, \mathbf{X} \sim Normal(\mathbf{0}, \sigma_u^2 \mathbf{I}_n)$$

with σ_u^2 nonzero and constant.

a) Please derive the statistical distribution of

$$(3\hat{\boldsymbol{\beta}} + 2\boldsymbol{\iota}) \,|\, \mathbf{X}$$

for which $\boldsymbol{\iota}$ is a (k+1) dimensional vector of ones.

b) We proceed with $\mathbf{u} \,|\, \mathbf{X}$. It is assumed that $\mathbf{u} \,|\, \mathbf{X} \sim Normal(\mathbf{0}, \boldsymbol{\Psi})$, for which $\boldsymbol{\Psi}$ is a symmetric matrix for which the inverse exists.

Question: Please derive the distribution of $\mathbf{u}'\boldsymbol{\Psi}^{-1}\mathbf{u} \,|\, \mathbf{X}$

Question 2)

For a random sample of n observations, the Ordinary Least Squares estimator $\hat{\boldsymbol{\beta}}_n$ is applied to the (k+1)-dimensional vector of parameters $\boldsymbol{\beta}$ of the linear regression model

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{u}$$

We consider the Central Limit Theorem, for which we formulate two additional assumptions.

1) $\frac{1}{n}\mathbf{X}'\mathbf{X} \xrightarrow{p} \mathbf{C}$ for which $\mathbf{C}$ is a finite and invertible (k+1) x (k+1) matrix.
2) $\frac{1}{\sqrt{n}}\sum_{i=1}^{n}\mathbf{x}_i u_i \xrightarrow{d} Normal(\mathbf{0}, \sigma_u^2\mathbf{C})$

Furthermore, it can be shown that $\sqrt{n}\left(\hat{\boldsymbol{\beta}}_n - \boldsymbol{\beta}\right) = \left(\frac{1}{n}\mathbf{X}'\mathbf{X}\right)^{-1}\frac{1}{\sqrt{n}}\sum_{i=1}^{n}\mathbf{x}_i u_i$ (you don't need to derive this expression).

a) Using the information of above, could you please derive the limiting statistical distribution of

$$\sqrt{n}\left(\hat{\boldsymbol{\beta}}_n - \boldsymbol{\beta}\right)$$

Question 3)

For a sample of *n* observations, for the linear regression model.

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{u}$$

let's assume that the variance covariance matrix of the error terms contains heteroskedasticity:

$$Var(\mathbf{u} \mid \mathbf{X}) = \text{diag}\left(\sigma_i^2\right) \qquad i = 1,\dots,n$$

a) Please, discuss the consequences of heteroskedasticity for the expected value of the OLS estimator $\hat{\boldsymbol{\beta}} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{y}$.

b) It can be demonstrated that $Var(\hat{\boldsymbol{\beta}} \mid \mathbf{X}) = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\boldsymbol{\Psi}\mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}$, for which $\mathbf{X}'\boldsymbol{\Psi}\mathbf{X} = \sum_{i=1}^{n} \mathbf{x}_i \sigma_i^2 \mathbf{x}_i'$. Please carefully describe both stages of the estimation procedure to calculate the White robust standard errors.

Question 4)

Let's consider the linear regression model

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{u}$$

for which the covariance matrix of the error terms is

$$\boldsymbol{\Psi} = Var(\mathbf{u} \mid \mathbf{X})$$

a) Derive the Generalized Least Squares estimator $\hat{\boldsymbol{\beta}}_{GLS}$ by minimizing the loss function

$$\min_{\boldsymbol{\beta}} L(\boldsymbol{\beta}) = \min_{\boldsymbol{\beta}} (\mathbf{y} - \mathbf{X}\boldsymbol{\beta})'\boldsymbol{\Psi}^{-1}(\mathbf{y} - \mathbf{X}\boldsymbol{\beta})$$

b) What assumptions are required for the existence of the GLS estimator $\hat{\boldsymbol{\beta}}_{GLS}$?

Question 5)
We consider the AR(1) model

$$x_t = \rho x_{t-1} + e_t \qquad |\rho| < 1 \qquad t = 1, \ldots, T$$

where e_t : i.i.d. (identically and independently distributed) with $Ee_t = 0$; $Var(e_t) = \sigma_e^2$. e_t is uncorrelated to x_{t-1}.

a) Please demonstrate that ρ can be interpreted as the correlation between x_t and x_{t-1}.

Question 6)
We consider the panel data model

$$y_{it} = \mathbf{x}_{it}'\boldsymbol{\beta} + \alpha_i + u_{it} \qquad i = 1, ..., n; t = 1, ..., T$$

for which α_i is the individual-specific effect (random variable) with constant variance, and u_{it} is the identically and independently distributed error term with expected value zero and constant variance.

a) Demonstrate that the assumption of strict exogeneity is required for the fixed-effects estimator.

b) Describe how the Mundlak's formulation of the regression equation

$$y_{it} = \mathbf{x}_{it}'\boldsymbol{\beta} + \alpha_i + u_{it} \qquad i = 1, ..., n; t = 1, ..., T$$

can be applied to test for a random effects specification (the zero hypothesis) versus fixed effects specification (the alternative hypothesis).

< end of the exam >