Utrecht University | School of Economics

Course name: Econometrics (Wisb377)
Date examination: October 17, 2019
Duration 2 hours; from <11:00>, to <13:00>
Examination: Midterm
Total number of pages: 4
Total number of exercises: 3

Full name :---

Student ID Number : ---

Copy your Name and ID Number on every separate sheet/answering paper if required

Signature : ---

- Put your ID on your table for inspection.

Exam instructions

At the start of the exam

- Candidates who arrive 30 minutes after the time scheduled for the start of the examination will not be permitted entry to the examination room.

During the examination

- Nobody is allowed to leave the room within the first 30 minutes after the start of the exam.
- You are not allowed to go to the restroom unless you have permission of the Examiner or an invigilator.
- **MOBILE PHONES AND OTHER COMMUNICATION DEVICES ARE ONLY ALLOWED WHEN SWITCHED OFF AND STORED IN CLOSED BAGS**.
- *It is a closed book exam. It is* ***not*** *allowed to use any study aids such as books, readers, (preprogrammed) calculators*
- You may use a simple calculator and a dictionary (without any [handwritten] notes in it).
- The exam form is **NOT** allowed to be taken home by the candidate

Results/Post-examination regulations:

- The results of the examination will be announced on Blackboard within two weeks of the exam date. At the same time the time & place of the exam inspection will be announced.
- We do not discuss exam results over the phone or by email.
- After the announcement of the exam results in OSIRIS you have four weeks within which to lodge an appeal against your grade.
- Four weeks after the results of this exam are published, the original exam is available to you, when a declaration is signed, stating that no appeal has been made or will be made.
- You can request a photocopy of your answers at the Student Desk up and until four weeks after publication of the results.

Questions

1) The vector of parameters $\boldsymbol{\beta}$ of the linear regression model

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{u}$$

is estimated by Ordinary Least Squares (OLS), using a sample of *n* observations.

$$\hat{\boldsymbol{\beta}} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{y}$$

We are interested in the assumptions that are needed to derive an unbiased estimator

$$E\left(\hat{\boldsymbol{\beta}} \mid \mathbf{X}\right) = \boldsymbol{\beta}.$$

a) Please give a careful explanation why it can be useful to have the assumption of a <u>randomly drawn</u> sample for an unbiased estimator. Motivate your answer by providing the proof for $E\left(\hat{\boldsymbol{\beta}} \mid \mathbf{X}\right) = \boldsymbol{\beta}$.

b) Next, it is assumed that the variance-covariance matrix of the vector of error terms is

$$Var(\mathbf{u} \mid \mathbf{X}) = \sigma_u^2 \mathbf{I}_n$$

Please derive the (*k*+1) x (*k*+1) variance-covariance matrix of $\hat{\boldsymbol{\beta}}$.

c) For $\hat{\mathbf{y}} = \mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{y}$ and $\hat{\mathbf{u}} = \mathbf{y} - \hat{\mathbf{y}}$, demonstrate that

$\hat{\mathbf{u}}'\hat{\mathbf{u}} = \mathbf{u}'\mathbf{M}_{\mathbf{X}}\mathbf{u}$ for which $\mathbf{M}_{\mathbf{X}} = \mathbf{I}_n - \mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'$

2)

a) For the regression equation

$$wage_i = \beta_0 + \beta_1 birthyear_i + \beta_2 year_i + \beta_3 age_i + u_i \qquad i = 1,\ldots,n$$

there is perfect multicollinearity because

$$year_i = birthyear_i + age_i$$

Please compute the R^2 of the auxiliary regression:

$$birthyear_i = \alpha_0 + \alpha_1 year_i + \alpha_2 age_i + v_i$$

for which

$$R^2 = 1 - \frac{\sum_{i=1}^{n} \hat{v}_i^2}{\sum_{i=1}^{n} (birthyear_i - \overline{birthyear})^2}$$

b) The dependent variable **y** is regressed on a vector of ones (with no further explanatory variables)

$$y_i = \beta + u_i \qquad i=1,\ldots,n \text{ with } Var(u) = \sigma_u^2$$

Compute the following for this case

- The Ordinary Least Squares estimator $\hat{\beta}$
- Compute $\hat{y}_i$ and $\hat{u}_i$ $i=1,\ldots,n$
- Compute $\hat{\sigma}_u^2$
- Compute R^2
- For $\mathbf{M_X} = \mathbf{I}_n - \mathbf{X}(\mathbf{X'X})^{-1}\mathbf{X'}$, compute $\mathbf{M_X}$ for $\mathbf{X} = \boldsymbol{\iota}$ and show that $\mathbf{M}_{\boldsymbol{\iota}}$ is a non-invertible, symmetric and idempotent matrix. Compute the trace of $\mathbf{M}_{\boldsymbol{\iota}}$.

c) For the linear regression equation

$$\ln(wage_i) = \beta_0 + \beta_1 education_i + \beta_2 age_i + \beta_3 age_i^2 + u_i$$

compute the partial effect of age on the wage at *age* = 20. Please, give a careful motivation of the assumption(s), for which the partial effect of age on the dependent variable can be interpreted as a causal effect?

3) For the OLS estimator $\hat{\boldsymbol{\beta}} = \left(\mathbf{X}'\mathbf{X}\right)^{-1}\mathbf{X}'\mathbf{y}$ of the linear regression equation $\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{u}$, for which $\mathbf{y}$ and $\mathbf{u}$ are n-dimensional vectors, $\boldsymbol{\beta}$ is a (k+1)-dimensional vector , $\mathbf{X}$ is a (n x (k + 1)) dimensional matrix,

c) Please give a careful description – included the assumptions – of the Strong Law of Large Numbers.

< end of the exam >